Math 1000 – Dalhousie University – Summer 2010

Assignment 3 – Due Wednesday July 14th

1. Use the rules of differentiation to find $f'(x)$ for each of the following functions.

   (a) $f(x) = \dfrac{x^3}{6} + \dfrac{6}{x^3}$

   (b) $f(x) = 3\sqrt[3]{x^5} + \dfrac{1}{\sqrt[4]{x^3}} - \dfrac{x^8}{10} + 3x - 29$

   (c) $f(x) = 2e^x \sec x$

   (d) $f(x) = \dfrac{\sin x}{x^2 e^x}$ (be careful – this will require both the quotient and product rule).

2. (a) Find the equation of the tangent line to $f(x) = x^{2/3}$ when $x = 8$.

   (b) At what point in the open interval $(-2\pi, 2\pi)$ does the tangent line to $f(x) = \sin x$ have slope 1? Find the solution using $f'(x)$ and justify by sketching the graph of $\sin x$.

3. Suppose you are riding your bike on a long hill, and the distance (in meters) you have travelled after t minutes can be determined by the formula $s(t) = \dfrac{t^3}{6} + 40t^2 - \dfrac{25t}{6}$.

   (a) How far have you travelled after 4 minutes? (Specify the units!)

   (b) How long does it take you to travel 1km (1000m)?

   (c) How fast are you riding once you reach the 1km mark?

   (d)What is your acceleration at t min? Do you think you're riding up or down the hill?

4. Using the fact that $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$ and the multiplicative rule $\lim\limits_{x\to a} f \cdot g = \left(\lim\limits_{x\to a} f\right)\left(\lim\limits_{x\to a} g\right)$, evaluate the following limits.

   (a) $\lim\limits_{\theta\to 0} \dfrac{\sqrt{\theta+1}\sin\theta}{2\theta}$

   (b) $\lim\limits_{t\to 0} \dfrac{\sin(t^2)}{t}$

5. Prove that $\frac{d}{dx}\tan x = \sec^2 x$ <u>without</u> directly using the definition of the derivative.

6. Given $f(x) = \cos x$, find $f^{(99)}(x)$ (the 99th derivative of $\cos x$).