Math 1000 – Dalhousie University – Summer 2010

Assignment 9 – Due Wednesday August 11

1. Use the definition of area under a curve to find an expression for the area under the graph of $f(x) = \dfrac{\ln x}{x}$ on the interval $[3, 10]$. Do not solve this limit (it will be ugly).

2. Determine a region whose area is equal to $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{2}{n}\left(5 + \dfrac{2i}{n}\right)^{10}$.

3. (a) Express $\int_1^4 3x^2\, dx$ as the limit of Riemann sums (with right endpoints); do not solve.

   (b) Express $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{\cos x_i}{x_i}\Delta x$ as a definite integral on the interval $[\pi, 2\pi]$; do not solve.

4. Evaluate the following definite integrals:

   (a) $\int_{-2}^{5} 6\, dx$.

   (b) $\int_1^8 \sqrt[3]{x}\, dx$.

   (c) $\int_0^{\pi/4} \sec\theta \tan\theta\, d\theta$.

   (d) $\int_1^2 \dfrac{4 + x^2}{x^3}\, dx$.

   (e) $\int_0^1 \left(1 + \dfrac{1}{2}u^4 - \dfrac{2}{5}u^9\right) du$.

   (f) $\int_0^1 10^x\, dx$. [Hint: Recall that $\frac{d}{dx}a^x = a^x \ln a$. So $\frac{1}{\ln a}a^x$ is an antiderivative of a^x.]

BONUS QUESTION:

Use part 1 of the Fundamental Theorem of Calculus to find $g'(x)$ if $g(x) = \int_1^{\sqrt{x}} t^3 dt$.