Assignment #5

Due: 1 pm, Friday, November 12, 2010

1. **(Multi-period investment strategy)** Bert Humphries has left his home in a small town in Manitoba and moved to Toronto to enroll in a four-year undergraduate program in economics. Bert is planning an investment strategy for the four-year period, with the goal of having the largest possible amount of cash on hand when he graduates. Bert has some revenue from a combination of an RESP (registered education savings plan) fund that his parents opened for him when he was a baby, scholarship funds, and a small inheritance he recently received from his grandmother. His estimate of revenues and expenses over the four years is shown in the following table. Bert has $5,000 in cash at the beginning of the first year. Assume that right now it is the beginning of Year 1, and both revenue and expense occur at the beginning of each year.

Year	Revenues	Expenses
1	$30,000	$10,000
2	$40,000	$25,000
3	$25,000	$25,000
4	$15,000	$12,500

Any money left over in each year can be invested in guaranteed investment certificates (GICs) for one year at a rate of 1.45%, for two years at 3.73%, for three years at 4.88%, or for four years at 6.98%. Bert uses the following rule for each of the four years:

Money invested + Expenses paid ≤ Revenue + Money earned from investment

   a) Formulate an LP model to determine an investment strategy that will maximize the value of Bert's GICs at the end of the fourth year.

   (Hint: Bert can have up to four GICs coming due at the end of the fourth year: a four-year GIC invested at the beginning of Year 1; a three-year GIC invested at the beginning of Year 2; a two-year GIC invested at the beginning of Year 3; and a one-year GIC invested at the beginning of Year 4. It is the sum of these values that Bert wants to maximize, as he plans to cash in all his GICs and travel the world for a year between graduation and starting his career.)

   b) Solve with Excel. Please clearly state the optimal investment strategy and the maximum cash on hand at the end of Year 4. Hand in an Excel output with the LP model, optimal solution and Sensitivity Report.

2. **(Capital budgeting)** Stockbroker Mary Bloom has made the following recommendation to a client:

Type of Investment	Cost ($)	Expected Return ($)
B.C. Bonds	500	50
Quebec City Bonds	1000	100
Saskatchewan Electricity	350	30
New Brunswick Gas	490	45
PEI Electric	700	65
Canada Paint Co.	270	20
Northern Hotel Co.	800	90
Nunavut Art Co.	400	35

The client agrees to this list, but provides several conditions: (1) no more than $3,000 can be invested; (2) the money is to be spread among at least five investments; (3) no more than one type of bond can be purchased; (4) at least two utility stocks must be purchased; (5) if Nunavut Art Co. is purchased, then Canada Paint Co. must be purchased, too.

   a) Formulate this as 0-1 IP model.
   b) Solve with Excel. Please clearly state which investments will be selected and the maximum expected return. Hand in an Excel output with the LP model and optimal solution (no Sensitivity Report).

3. **(Sensitivity Analysis, the Office Manager's File Cabinet)** The problem was formulated as an LP model in A2, and solved using both graphical method and Excel Solver in A3. The results are shown below (without the ratio constraint).

Suppose A = number of Ace file cabinets to purchase, and E = number of Excello file cabinets to purchase.

Max Filing Capacity $= 24A + 36E$

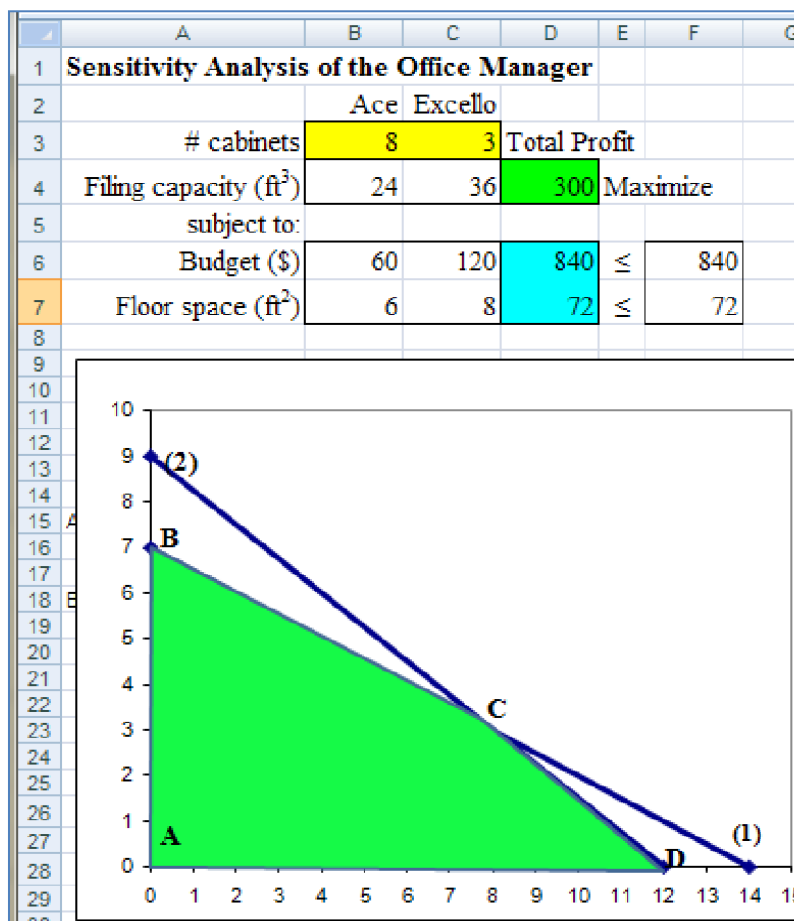*Subject to*:

$$60A + 120E \leq 840 \quad (0, 7) \ (14, 7)$$
$$6A + 8E \leq 72 \quad\quad (0, 9) \ (12, 0)$$
$$A \geq 0, E \geq 0$$

Corner	Coordinates	Filing Capacity $= 24A + 36E$
A	(0, 0)	0
B	(0, 7)	252
C	**(8, 3)**	**300**
D	(12, 0)	288

The optimal solution is the corner C, where $A = 8$ & $E = 3$, and max FC = 300.

That is, the manager should buy eight Ace cabinets and three Excello cabinets, and the maximum filing capacity is 300 cubic feet.

Use the graphic method to answer the following questions.

a) Consider the 1st constraint (budget), $60A + 120E \leq 840$, where an Ace cabinet costs \$60, an Excello cabinet costs \$120, and the total budget is \$840.

   i) What are the new optimal solution and the maximum filing capacity if the budget increases to \$960? That is, the RHS of the constraint increased by \$120 ($60A + 120E \leq 960$).

   ii) What is the shadow price of the budget constraint?

   iii) What is the RHS Range? (Within this range the shadow price remains the same.)

b) Consider the 2nd constraint of floor space, $6A + 8E \leq 72$, where an Ace cabinet requires 6 ft^2 of floor space, an Excello cabinet requires 8 ft^2 of floor space, and there is no more than 72 ft^2 floor space for the file cabinets.

   i) What are the new optimal solution and the maximum filing capacity if the floor space increases to 78 ft^2? That is, the RHS of the constraint increased by 6 ft^2 ($6A + 8E \leq 78$).

   ii) What is the shadow price of the constraint?

   iii) What is the RHS Range? (Within this range the shadow price remains the same.)

c) What is the highest and the lowest filing capacity per unit for Ace cabinet? That is, find the Allowable Range for the objective function coefficient (OFC) of A.

d) Run Excel Solver again and hand in an output of the LP model and the sensitivity report. Use your computer output and check your calculation results for (a) to (c).

4. **(Fran Farnsworth)** Fran Farnsworth travels throughout the Maritimes selling a line of three health care products. Each box of Flex costs $3 from her supplier, takes Fran 10 minutes (on average) to sell, costs $0.50 to deliver, and contributes $2.50 to Fran's commission. Each box of Feelgood costs $5, takes 20 minutes to sell, is left with the customer at the time of sale, and contributes $5 to Fran's commission. Each box of Firmup costs $4, takes 12 minutes to sell, costs $1 to deliver, and contributes $3.50 to Fran's commission. Every week Fran is allowed to draw up to $1200 worth of products, and has a weekly delivery expense allowance of $90. Due to travel time and other commitments, Fran is limited to 42 hours, or 2520 minutes, per week for sales.

Suppose x, y, & z represent the number of boxes to sell per week of Flex, Feelgood and Firmup respectively; and C is Fran's weekly commissions. Then the LP model is:

$$Max\ Commission = 2.5x + 5y + 3.5z$$

$$
\begin{array}{lll}
\text{s.t.} & 3x + 5y + 4z \le 1{,}200 & (\$-worth) \\
 & 10x + 20y + 12z \le 2{,}520 & (min-sell\ time) \\
 & 0.5x \quad + \quad z \le 90 & (\$-delivery\ cost)
\end{array}
$$

$$All\ variables\ are\ non-negative.$$

a) Set up this model in Excel. Solve the LP model using Excel Solver. Print a one-page Excel output with the LP model and Sensitivity Report.

Use the information given in the Sensitivity Report to answer the following questions.

b) How many boxes of each health care product should Fran sell per week, and what is her maximum weekly commission?

c) Given the optimal solution, how much time does Fran spend each week to sell the health care products? How much does she spend in her delivery allowance?

d) Should Fran ask her boss for more than the $1200 worth of products she is allowed to sell per week, assuming no other conditions change? Explain.

e) If Fran can hire a baby sitter for Friday afternoon at $10 per hour, this would allow her to increase her weekly selling time to 46 hours instead of 42 hours. Should she do it? If so, how specifically would her selling decisions and commission change?

f) How would Fran's selling decision and commission change if her delivery expense allowance were reduced to $50?

g) A recent sales promotion by her supplier has pressed Fran to sell at least 10 boxes of Flex per week. If she does that, how much will her weekly commission decrease?

h) For each product, if their profits (commission contribution) were higher, Fran might wish to sell more of them. How high would the unit profits have to be to make Fran change her decision to sell? Lower profits would make products less attractive. How low would they have to be before Fran would decrease the number of boxes to sell? Fill in the table with your answer.

	Profit Range (per box)		
	Lower Limit	Current Value	Upper Limit
Flex		$2.2	
Feelgood		$5	
Firmup		$3.5	