

Solution to Assignment #1**Due:** 1 pm, Friday, September 24 2010

- # 1. The management of **the Toy R4U** Company needs to decide whether to introduce a certain new novelty toy for the upcoming Christmas season, after which it would be discounted. The total cost required to setup the production and market this toy would be \$500,000 plus \$15 per toy produced. The company would receive revenue of \$35 for each toy sold.

The decision Variable is x = number of toys to produce.

$$\text{Fixed cost } F = \$500,000$$

$$\text{Variable cost per unit } c = \$15$$

$$\text{Sell price per unit } p = \$35$$

- (a) Assume that company could sell every unit of this toy produced. Write an expression for the Total Cost (C), Revenue (R), and Profit (P) functions for the company in terms of x , the number of units produced.

$$\text{The cost function: } C(x) = cx + F = 15x + 500,000$$

$$\text{The revenue function: } R(x) = px = 35x$$

$$\text{The profit function: } P(x) = R(x) - C(x) = (p - c)x - F = 20x - 500,000$$

- (b) How many units must be produced and sold in order to break even?

$$\begin{aligned} \text{To break even, we have } P(x) = 0 &\Rightarrow P(x) = 20x - 500,000 = 0 \\ 20x &= 500,000 & x &= 25,000 \end{aligned}$$

Therefore, the Toy R4U should make 25,000 toys to break even.

$x = 25,000$ is called the breakeven point (BEP).

- (c) What is the company's break-even point in dollars (BEP_{\$})? That is, how much is the company's total cost, revenue and profit at the break-even production level?

$$\text{When breakeven, } x = 25,000. \text{ Thus, } R(25,000) = C(25,000) = 35 \times 25,000 = \$875,000$$

Therefore, when $x = 25,000$, the total cost equals the revenue, equals \$875,000.

$R = C = \$875,000$ is also called the breakeven point in dollars (BEP_{\$}).

- (d) Find the total cost, revenue and profit when 35,000 toys are made and sold.

When $x = 35,000$

$$\text{The cost: } C(35,000) = 15 \times 35,000 + 500,000 = 1,025,000$$

$$\text{The revenue: } R(35,000) = px = 35 \times 35,000 = 1,225,000$$

$$\begin{aligned} \text{The profit: } P(35,000) &= R(x) - C(x) = 1,225,000 - 1,025,000 = 200,000 \text{ or} \\ P(35,000) &= 20x - 500,000 = 20 \times 35,000 - 500,000 = 200,000 \end{aligned}$$

Therefore, when 35,000 toys are made and sold, the total cost would be \$1,025,000, the revenue \$1,225,000, and the profit \$200,000.

- (e) If the company wants to make a profit of a million dollars, how many units must be made and sold?

$$\text{We want } P = 1,000,000: P(x) = 20x - 500,000 = 1,000,000$$

$$20x = 1,500,000 \quad x = 75,000$$

To make a million dollar profit, the company has to produce and sell 75,000 toys.

- (f) How many toys should be made and sold if the company wants its profit to be at least 35% of the revenue?

$$\text{Target } P = 0.35 \times R(x) = 0.35(35x) = 12.25x$$

$$P(x) = 20x - 500,000 = 12.25x$$

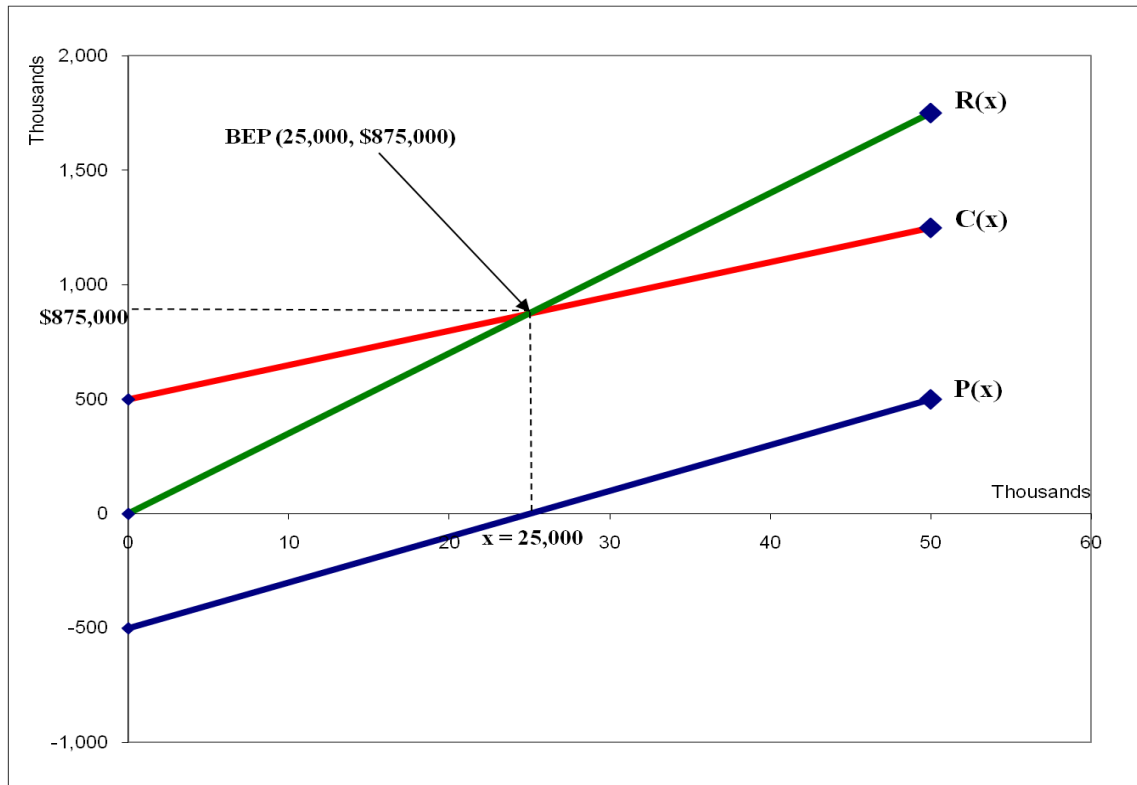
$$20x - 12.25x = 500,000$$

$$7.75x = 500,000$$

$$x = 64,517$$

To make a profit at least 35% of the revenue, the company has to produce and sell at least 64,517 toys.

- (g) Sketch a graph of the cost, revenue, and profit functions, and identify the break-even point. For the BEP point, clearly indicate the number of toys produced, and the corresponding cost and revenue in dollars.



- (h) Set up an Excel model for this problem, and use Goal Seek to find the BEP. Attach a hard copy of your Excel result to your assignment. You need to indicate the **Set cell**, **To value**, and **By changing cell**, which you used in your Goal Seek window.

	A	B	C	D	E
1	#1 Toy R4U				Name & ID
2	Fixed cost	500,000	\$		
3	Variable cost	15	\$ per unit		
4	Selling price	35	\$ per unit		
5					
6	# units	25,000			
7	Total Cost	875,000			
8	Revenue	875,000			
9	Profit	0			
10					

Goal Seek

Set cell: E6

To value: 0

By changing cell: \$B\$6

OK Cancel

#2. The Special Products Company produces expensive and unusual gift to be sold in specialized stores. The latest new-product proposal suggests a limited edition grandfather clock. The management needs to decide whether to introduce this product and, if so, how many of these grandfather clocks to produce. If the company does go ahead with this product, a fixed cost of \$50,000 would be incurred for setting up the production facilities to produce the product. It costs \$400 for each clock produced. Each clock would generate revenue of \$900 for the company.

The decision Variable is x = how many grandfather clocks to produce.

$$\text{Fixed cost } F = \$50,000$$

$$\text{Variable cost per unit } c = \$400$$

$$\text{Sell price per unit } p = \$900$$

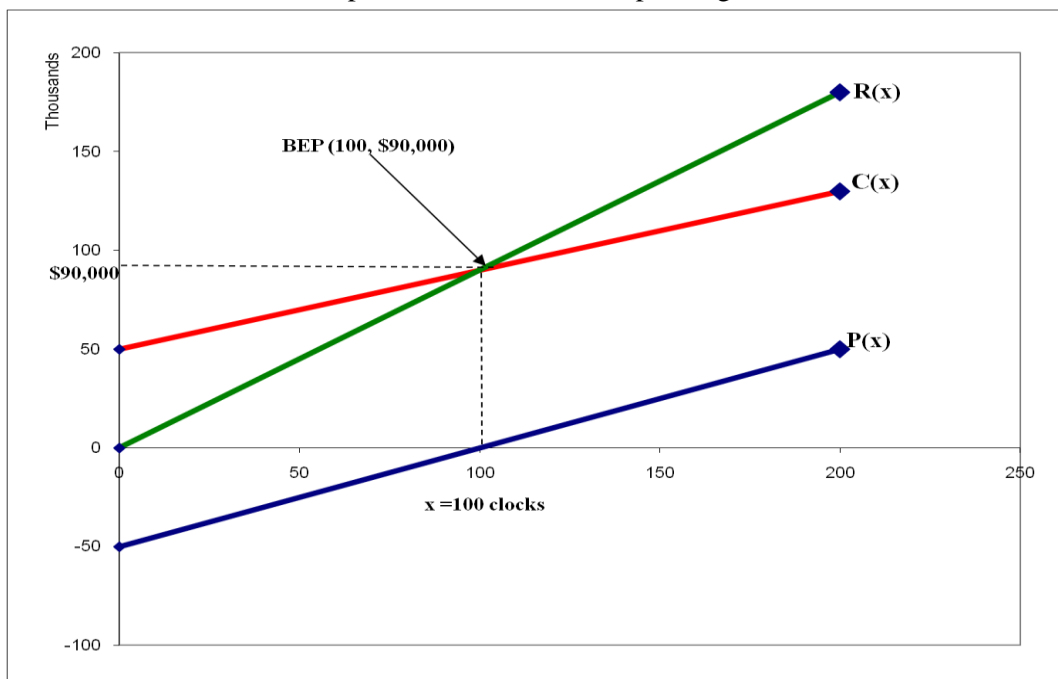
- (a) Determine the Total Cost (C), Revenue (R), and Profit (P) functions, in terms of the number of clocks made and sold.

$$\text{The cost function: } C(x) = cx + F = 400x + 50,000$$

$$\text{The revenue function: } R(x) = px = 900x$$

$$\text{The profit function: } P(x) = R(x) - C(x) = (p - c)x - F = 500x - 50,000$$

- (b) Sketch a graph of the cost, revenue, and profit functions, and identify the break-even point. For the BEP point, clearly indicate the number of clocks produced, and the corresponding cost and revenue in dollars.



- (c) If the company anticipates that they could sell 300 units, how large can the setup cost be before the grandfather clocks cease to be profitable?

To breakeven when $x = 300$, $F = ?$

$$P(x) = (p - c)x - F = 0, \quad \text{where } p = 900, c = 400, \text{ and } x = 300, \text{ find } F.$$

$$(900 - 400) \times 300 - F = 0$$

$$150,000 - F = 0$$

$$F = 150,000$$

Therefore, the highest setup cost is \$150,000. When the setup cost is higher than \$150,000, the profit would be negative.

- (d) If the company anticipates that they could sell 300 units, how large can the production cost per unit be before the grandfather clocks cease to be profitable?

To breakeven when $x = 300$, $c = ?$

$P(x) = (p - c)x - F = 0$, where $p = 900$, $F = 50,000$, and $x = 300$, find c .

$$(900 - c) \times 300 - 50,000 = 0$$

$$(900 - c) \times 300 = 50,000$$

$$270,000 - 300c = 50,000$$

$$300c = 220,000$$

$$c = 733.33$$

Therefore, the highest variable cost per unit is \$733.33. When the cost per unit goes higher than \$733.33, the profit would be negative.

- (e) If the company anticipates that they could sell 300 units, how small can the selling price per unit be before the grandfather clocks cease to be profitable?

To breakeven when $x = 300$, $F = ?$

$P(x) = (p - c)x - F = 0$, where $F = 50,000$, $c = 400$, and $x = 300$, find p .

$$(p - 400) \times 300 - 50,000 = 0$$

$$300p - 120,000 - 50,000 = 0$$

$$300p = 170,000$$

$$p = \$566.67$$

Therefore, the lowest selling price is \$566.67. For any price lower than this value, the profit would be negative.

- (f) When you answer the questions (c) through (e), which variable is the decision variable? Which are model parameters?

Question	Decision Variable	Model Parameters
(c)	F	x, c, p
(d)	c	x, F, p
(e)	p	x, F, c

- #3. The Power Notebooks, Inc. produces and sells a new type of LED monitors. The company estimates the demand and supply functions for the product are linear functions as given below, where p is the price per unit and q is the quantity sold or supplied.

$$D: q = -10p + 11,000$$

$$S: q = 12.5p - 2,500$$

- (a) Determine the equilibrium quantity and price.

$$-10p + 11,000 = 12.5p - 2,500$$

Substitute $p = \$600$ into D or S:

$$-22.5p = -13,500$$

$$q = 12.5(600) - 2,500$$

$$p = 600$$

$$q = 5,000$$

Therefore, the market equilibrium price is \$600 per unit, and 5,000 units could be sold at this price.

- (b) If customers are charged a 15% sales tax on the product, find the new demand function, which relates the before-tax unit price to the quantity sold.

The new demand function D' : $q = -10(1.15p) + 11,000 = -11.5p + 11,000$

- (c) Determine the new market equilibrium quantity and price resulting from the shift in demand found in (b). What is the tax amount charged on each unit, and what is total amount a customer has to pay for a unit?

$$-11.5p + 11,000 = 12.5p - 2,500 \quad \text{Substitute } p = \$562.5 \text{ into } D' \text{ or } S:$$

$$-24p = -13,500$$

$$q = 12.5(562.5) - 2,500$$

$$p = 562.50$$

$$q = 4,531.25 \approx 4532$$

Therefore, the market equilibrium price is \$562.5 per unit, and 4,532 units could be sold at this price.

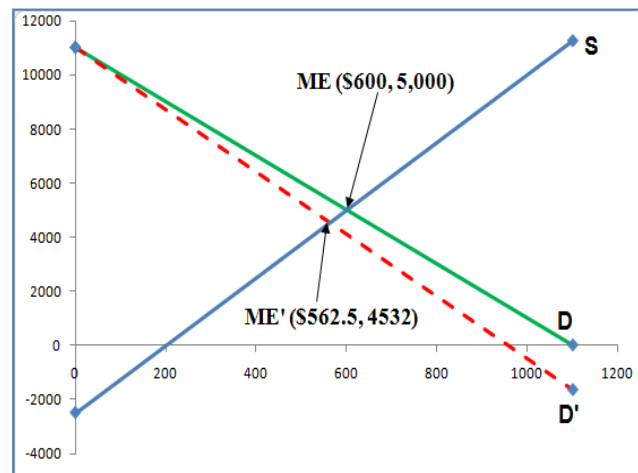
before-tax price	562.50	\$ price charged
Tax amount	84.38	\$ tax amount
After-tax price	646.88	\$ total amount paid

- (d) Compare the two market equilibriums. Which market equilibrium price is lower? Does it mean that the consumer will pay less with the lower price per unit? How much revenue could the produce make in either case? What causes the difference in revenue?

At the Market Equilibrium	No sales tax	With 15% sales tax
Price	\$600	\$562.50
Quantity	5,000 units	4,532 units
The amount consumer pays	\$600	\$646.88
The revenue supplier realizes	$600 \times 5000 = \$3,000,000$	$562.50 \times 4532 = \$2,549,250$

The effect of taxation: although the equilibrium is lower by \$37.5, the total amount the consumer actually pays is \$46.88 higher. On the supplier side, the revenue drops by \$450,750 due to the lower price and quantity.

- (e) Sketch a graph showing the supply function and both demand functions (with and without a sales tax). Also on the graph, indicate the market equilibrium point found in (a) and in (c).



(e) Set the problem in Excel. Find the market equilibrium in (a) using Goal Seek. Attach a hard copy of your result.

	A	B	C	D	E	F	G	H
1	#3. Power Notebooks, Inc.							
2	a) Basic ME							
3	Price per unit	600.00	\$ <- Changing cell					
4		m	b	# units q	Difference			
5	Demand no tax	-10	11000	5000	0			
6	Supply	12.5	-2500	5000				
7								
8								

Goal Seek

Set cell: G5\$E\$5

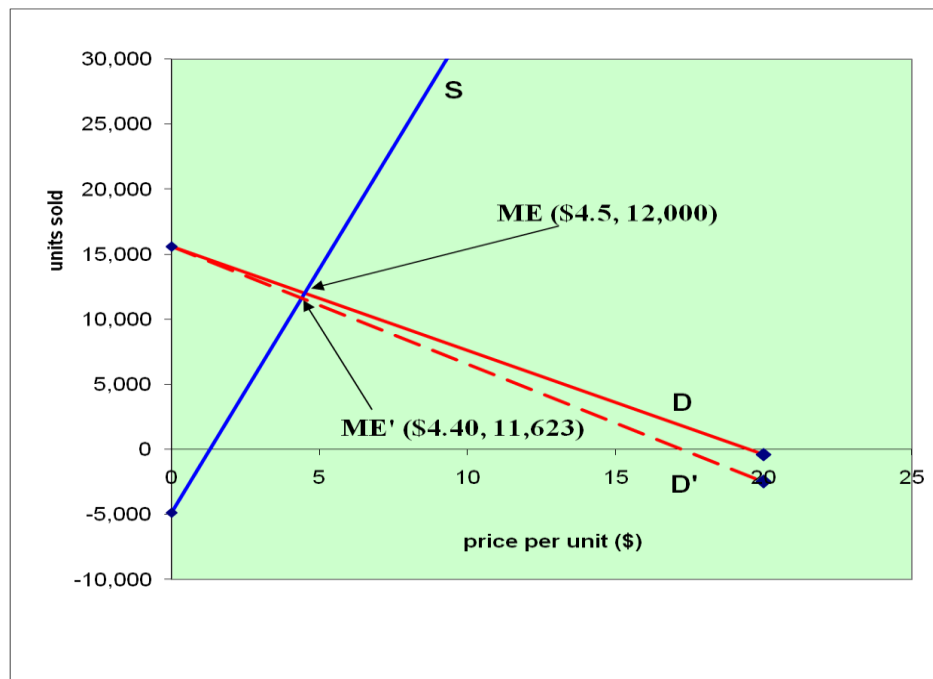
To value: 0

By changing cell: \$B\$3

OK Cancel

#4. The market equilibrium for a product occurs when 12,000 units are produced at a price of \$4.50 per unit. The producer will supply no units at \$1.30, and the consumers will demand no units at \$19.5.

(a) Sketch a graph showing the supply and demand functions, and indicate the points given in the question.



- (b) Assume both supply and demand relationships are linear. Determine the demand and supply functions for the product.

Price per unit	\$4.50	\$19.5	\$1.30
Demand	12,000	0	-
Supply	12,000	-	0

The demand function: $m = \frac{(q_2 - q_1)}{(p_2 - p_1)} = \frac{(12,000 - 0)}{(4.5 - 19.5)} = \frac{12,000}{15} = -800$

Substitute (\$19.5, 0) into $q = -800p + b$

$$0 = -800(19.5) + b$$

$$b = 800(19.5) = 15,600$$

Therefore, the demand function is $q = -800p + 15,600$

The supply function: $m = \frac{(q_2 - q_1)}{(p_2 - p_1)} = \frac{(12,000 - 0)}{(4.5 - 1.3)} = \frac{12,000}{3.2} = 3,750$

Substitute (\$1.30, 0) into $q = 3,750p + b$

$$0 = 3,750(1.3) + b$$

$$b = -4,875$$

Therefore, the demand function is $q = 3,750p - 4,875$

- (c) If customers are charged a 13% sales tax on the product, find the new demand function, which relates the before-tax unit price to the quantity sold.

The new demand function D': $q = -800(1.13p) + 15,600 = -904p + 15,600$