Solution Key to Assignment #2

Due: 1 pm, Friday, October 1, 2010

#1. For each of the following LP problems, find the optimal solution using the graphical method.

(a) Maximize $P = 2x + 2y$

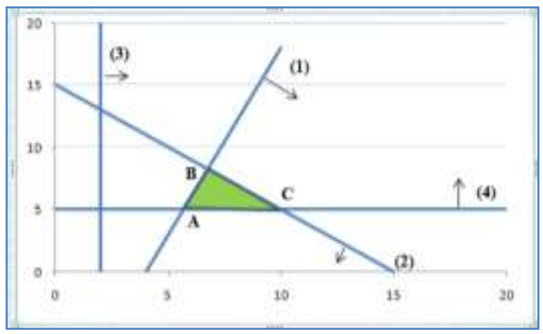
Subject to

$$3x - y \geq 12$$
$$x + y \leq 15$$
$$x \geq 2$$
$$y \geq 5$$

i) Identify the feasible solution region, S (see the graph below).

To draw a line for each of the constraints, we need to find two points:

	Constraint	point I	point II
1	$3x - y = 12$	(0,−12)	(4, 0)
2	$x + y = 15$	(0, 15)	(15, 0)
3	$x = 2$	(2, 0)	(2, 20)
4	$y = 5$	(0, 5)	(20, 5)



ii) Find the coordinates for all the corner points of S.

There are three corners in the feasible solution region, of which the coordinates of A and C could be found easily. Because $y = 5$ in both cases, therefore, $x = 5.67$ for corner A (put $y = 5$ in line (1)); and $x = 10$ for corner C (put $y = 5$ in line (2)).

Corner B: Solve constraints (1) & (2) together:

Add (1) & (2)

(1) $3x - y = 12$ → $4x = 27$

(2) $x + y = 15$ $\quad x = 6.75$

Corner B: (6.75, 8.25)

substitute $x = 6.75$ in (2)

$y = 8.25$

Corner	(x, y)	$P = 2x + 2y$
A	(5.67, 5)	$P = 2(5.67) + 2(5) = 21.34$
B	**(6.75, 8.25)**	$\boldsymbol{P = 2(6.75) + 2(58.25) = 30}$
C	(10, 5)	$\mathbf{P = 2(10) + 2(15) = 30}$

Therefore, there are infinitely many **optimal solutions, that is, any point on the segment between Corner B (*x = 6.75, y = 8.25*) and Corner C (*x = 10, y = 5*) *is an optimal solution*. They all yield the maximum profit of $30**.

(b) Minimize C = 4x + 2y

Subject to

$x + y \geq 10$

$5x + 2y \geq 32$

$-x + 2y \geq 0$

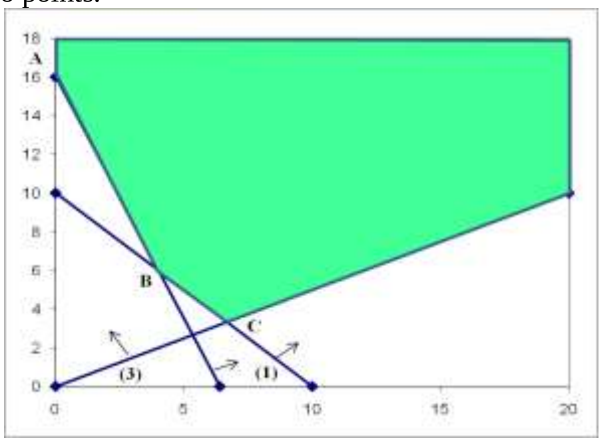
$x \geq 0$

$y \geq 0$

1) Identify the feasible solution region, S (see the graph below).

To draw a line for each of the constraints, we need to find two points:

	Constraint	point I	point II
1	$x + y = 10$	(0, 10)	(10, 0)
2	$5x + 2y = 32$	(0, 16)	(6.4, 0)
3	$-x + 2y = 0$	(0, 0)	(10, 5)



2) Find the coordinates for all the corner points of S.

There are three corners in the feasible solution region S. Of which, the coordinates of A may be obtained directly from the graph.

Corner B: Solve constraints (1) & (2) together: <u>Corner B: (4, 6)</u>

(1)*2 + (2)

$-2x - 2y = -20$

$5x + 2y = 32$

$3x \quad = 12 \qquad x = 4 \qquad y = 6$

For corner C: solve (1) and (3) together: <u>Corner C: (6.67, 3.33)</u>

(1) $x + y = 10 \qquad 3y = 10 \qquad y = 3.33$

(3) $x = 2y \qquad x = 6.67$

Corner	(x, y)	C = 4x + 2y
A	(0, 16)	C = 2(16) = 32
B	**(4, 6)**	**C = 4(4) + 2(6) = 28**
C	(6.67, 3.33)	C = 4(6.67) + 2(3.33) = 33.34

Therefore, the **<u>minimum cost is $28, and the optimal solution is *x* =*4* and *y* = *6*, which is the corner B</u>.**

#2. Formulate an LP model for each of the following problems. Please define the decision variables and list all the relevant constraints. **DO NOT SOLVE.**

a) An office manager needs to purchase new filling cabinets. She knows that each Ace cabinet costs \$60, requires 6 square feet of floor space, and holds 24 cubic feet of files. On the other hand, each Excello cabinet costs \$120, requires 8 square feet of floor space, and holds 36 cubic feet of files. Her budget permits her to spend no more than \$840 on cabinets, while the office has space for no more than 72 square feet of cabinets. The manager desires the greatest storage capacity within the limitations imposed by funds and space.

Formulate an LP model for the problem, which determines how many Ace cabinets and Excello cabinets should be purchased so that storage capacity is maximized.

	Ace	Excello	Limited Resources
Cost	\$60	\$120	No more than \$840
Space	6 ft^2	8 ft^2	No more than 72 ft^2
Filing capacity	24 ft^3	36 ft^3	<= Maximize

Suppose the office manager will use A units of Ace cabinets, and E units of Excello cabinets.

$Max\ \ File\ capacity = 24x + 36y$

$Subject\ to$:

$$60A + 120E \leq 840$$

$$6A + 8E \leq 72$$

$$A \geq 0, E \geq 0$$

b) Seall Manufacturing Company makes DVD players and Blue-Ray players. On the assembly line, each DVD player requires 5 hours, and each Blue Ray player takes 3 hours. Testing Department spends 2 hours on each DVD player, and 3 hours on each Blue Ray player. Both require 1 hour for Shipping and Packaging. On a particular production run, the company has available 3,900 hours on the assembly line, 3,000 hours in the Testing Department, and 2,000 hours in the Shipping and Packaging Department. The DVD player is sold for \$89 and the Blue Ray for \$129.

The management at Seall would like to know how many units of each product they should make so that the total revenue would be maximum. Formulate an LP model for the problem.

	DVD	Blue-Ray	Limited Resources
Assembly Time	5	3	3,900 hours
Testing Time	2	3	3,000 hours
Shipping	1	1	2,000 hours
Profit per unit	\$89	\$129	<= Maximize

Suppose Seall will make x units of DVD players and y units of Blue-Ray players.

$Max\ \ P = 89x + 129y$

$Subject\ to$:

$$5x + 3y \leq 3900$$

$$2x + 3y \leq 3000$$

$$x + y \leq 2000$$

$$x \geq 0, y \geq 0$$

c) Topgrade Turf lawn seed mixture contains three types of seeds: bluegrass, rye, and Bermuda. The costs per pound of the three types of seed are 18 cents, 22 cents and 9 cents, respectively. In each batch there must be at least 20% bluegrass seed, and the amount of Bermuda must be no more than 2/3 the amount of rye. To fill current orders, the company must make at least 5,000 pounds of the mixture. How much of each kind of seed should be used to minimize cost?

Formulate an LP model for the problem.

	Bluegrass	Rye	Bermuda	
Cost (cents)	18	22	9	<= Minimize
Total weight	1	1	1	At least 5,000 lbs
Bluegrass	At least 20%			
Bermuda vs. Rye			No more than 2/3 of rye	

Suppose the company will produce the mixture with x lbs of bluegrass; y lbs of rye and z lbs of Bermuda.

$Min\ \ Cost = 18x + 22y + 9z$ or: $Min\ \ Cost = 18x + 22y + 9z$

$Subject\ to$:

$$x + y + z \geq 5000$$
$$x \geq 0.2(5000) \qquad x \geq 1000$$
$$z \leq \frac{2}{3}y \qquad 2y - 3z \geq 0$$
$$x \geq 0, y \geq 0,\ z \geq 0$$

$Subject\ to$:

$$x + y + z \geq 5000$$
$$x \geq 0.2(x + y + z) \quad \rightarrow \quad 0.8x - 0.2y - 0.2z \geq 0$$
$$z \leq \frac{2}{3}y \quad \rightarrow \quad 2y - 3z \geq 0$$
$$x \geq 0, y \geq 0,\ z \geq 0$$

d) Susan Williams has decided to invest a $100,000 inheritance in government securities that earn 7% per year, municipal bonds that earn 6% per year, and mutual funds that earn 10% per year. She will spend at least $40,000 on government securities, and she wants at least half of the inheritance to go to bonds and mutual funds. Government securities have an initial fee of 2%, municipal bonds have an initial fee of 1%, and mutual funds have an initial fee of 3%. Susan has $2,400 available to pay initial fees. How much should be invested in each way to maximize the interest yet meet the constraints?

	Government Security	Municipal Bonds	Mutual Funds	Limited Resource
Total heritance	1	1	1	No more than $100,000
Return	7%	6%	10%	<= Maximize
Initial fees	2%	1%	3%	No more than $2400

Suppose Susan will invest x dollars in government security; y dollars in municipal bonds, and z dollars in mutual funds.

$Max\ Return = 0.07x + 0.06y + 0.1z$ or: $Max\ Return = 0.07x + 0.06y + 0.1z$

$Subject\ to$:

$$x + y + z \leq 100{,}000$$
$$0.02x + 0.01y + 0.03z \leq 2400$$
$$y + z \geq 50{,}000$$
$$x \geq 40{,}000$$
$$y \geq 0, z \geq 0$$

$Subject\ to$:

$$x + y + z \leq 100{,}000$$
$$0.02x + 0.01y + 0.03z \leq 2400$$
$$y + z \geq 0.5(x + y + z) \ \rightarrow\ -x + y + z \geq 0$$
$$x \geq 40{,}000$$
$$y \geq 0, z \geq 0$$