Solution Key to Assignment #3

Due: 1:00 pm, Friday, October 15, 2010

#1. $Max\ \ File\ capacity = 24A + 36E$ maximize total filing capacity

$Subject\ to$: $60A + 120E \leq 840$ cost

$6A + 8E \leq 72$ floor space

$A \geq 0, E \geq 0$

The LP model is formulated for #2 (a) in A2, where A & E represent number of units of Ace and Excello cabinets to be purchased, respectively.

a) Suppose that the office manager would like to purchase at least one Ace cabinet for every two Excello purchased. Write a constraint for this requirement.

$2A \geq E$, therefore, $2A - E \geq 0$

b) Add the constraint in part (a) to the LP model, and find the optimal solution using the graphical method. With the optimal solution, how many cabinets of each type should be purchased? What is maximum total filing capacity?

$Max\ \ File\ capacity = 24A + 36E$

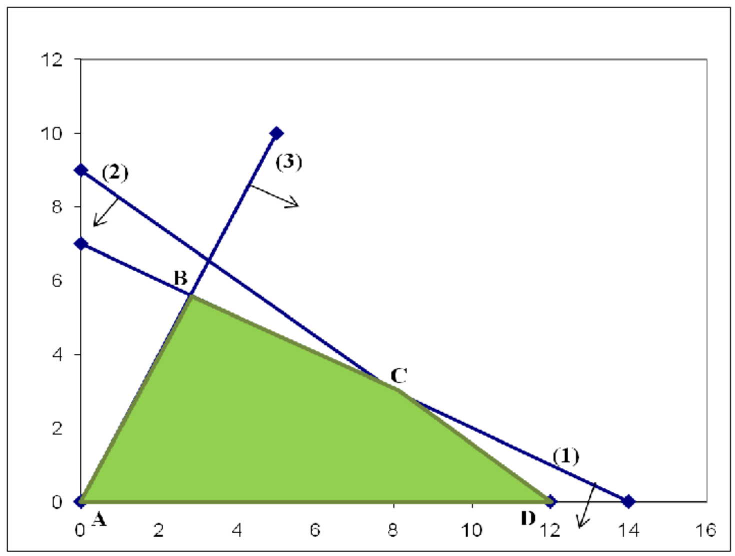
$Subject\ to$: $60A + 120E \leq 840$ (0, 7) (14, 7)

$6A + 8E \leq 72$ (0, 9) (12, 0)

$2A - E \geq 0$ (0, 0) (6, 12)

$A \geq 0, E \geq 0$

Corner	Coordinates	$FC = 24A + 36E$
A	(0, 0)	0
B	(2.8, 5.6)	268.8
C	**(8, 3)**	**300**
D	(12, 0)	288



Therefore, the optimal solution is the corner C, where *A = 8 & E = 3, and Max FC = 300.*

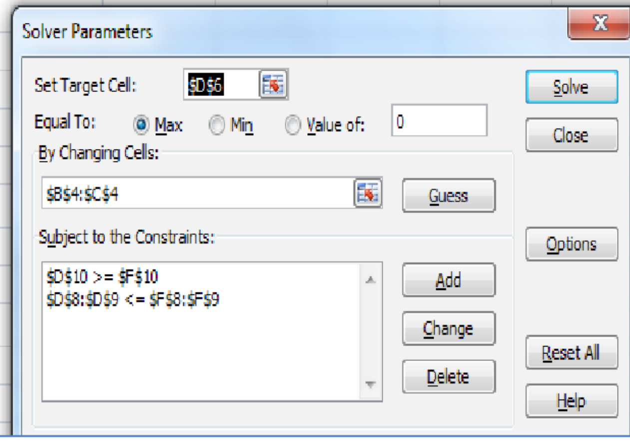
That is, the office manager should purchase 8 Ace file cabinets and 3 Excello cabinets. This way the total filing capacity is 300 cubic feet, which is the largest given the restrictions on budget and floor space.

c) With the optimal solution, what is the floor space used? How much of the budget remains unspent?

Because the Budget and the Floor Space are constraints (1) and (2), which are active at Corner C, therefore, both limited resources are used up. At the optimality, therefore, floor space used is 72 square feet, and budget used is $840. There is unspent budget.

d) Implement the LP model in Excel, and use Solver to find the optimal solution. Please follow the instructions given at the end of this assignment for Excel/Solver setups and output.

	A	B	C	D	E	F
1	**#2 (a) Office Manager**					
2		Ace	Excello			
3		x	y			
4	# cabinets	8	3			
5				Total Cost		
6	Filing capacity (ft^3)	24	36	300	Maximize	
7	subject to:					
8	Budget ($)	60	120	840	≤	840
9	Floor space (ft^2)	6	8	72	≤	72
10	Ratio	2	-1	13	≥	0
11						



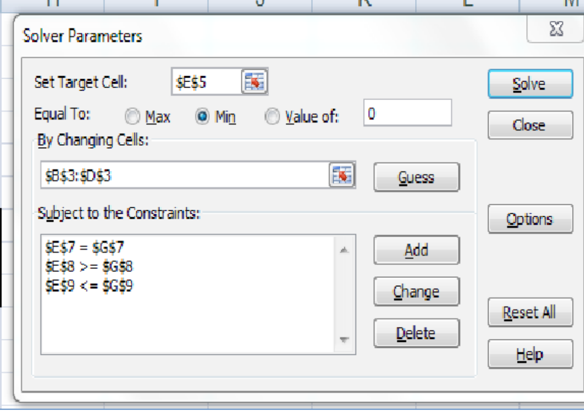
#2. $Min\ \ Cost = 18x + 22y + 9z$

$$Subject\ to{:}\ \ x + y + z \geq 5000$$
$$x \geq 1000$$
$$2y - 3z \geq 0$$
$$x \geq 0, y \geq 0,\ z \geq 0$$

The LP model is formulated for A2 #2 (c), where x lbs of bluegrass; y lbs of rye and z lbs of Bermuda are used in the mixture.

a) Implement the LP model in Excel, and use Solver to find the optimal solution. Please follow the instructions given at the end of this assignment for Excel/Solver setups and output.

	A	B	C	D	E	F	G
1	**#2 c) Topgrade Turf Lawn**						
2		Bluegrass	Rye	Bermuda			
3	# lbs used	1000	2400	1600			
4					Total Cost		
5	Cost (cents)	18	22	9	85200	Minimize	
6	subject to:						
7	Total weight	1	1	1	5000	=	5,000
8	Lower limit on bluegrass	1			1000	≥	1,000
9	Bermuda vs. Rye		-2	3	0	≤	0
10	Therefore, the optimal solution is to use 1000 pounds of Bluegrass, 2400 pounds of rye,						
11	and 1600 pounds of Bermuda. Thus, 5000 pounds of mixture could be made						
12	with the minimum cost for $852.						



b) In the optimal mixture, how many pounds of each type of seeds will be used? What is the minimum cost? For the mixture, what is the minimum cost per pound?

The optimal solution and the min cost have been given in the Excel output.

The minimum cost per pound = $\frac{852}{5000} = 0.1704$, that is, \$0.17 per pound.

#3. A manufacturer would like to produce a total of 300 ceramic tiles, which are available in three styles. The first requires 5 minutes of shaping, and no painting time; the second style requires 2 minutes of shaping and 3 minutes of painting; and the third style requires 2 minutes of shaping and 5 minutes of painting. There are at most 25 hours available for shaping, which costs $18 per hour (labor and machine); and at most 20 hours for painting, which costs $12 per hour. The selling prices on the three styles are $3, $4 and $5 per tile, respectively. How many tiles of each style should be produced for a maximum profit? **Formulate an LP model for the problem. DO NOT SOLVE.**

	Style #1	Style #2	Style #3	Availability/requirement
# tiles	1	1	1	Need 300
Shaping	5 min	2 min	2 min	25 hrs at $18 per hour
Painting	0 min	3 min	5 min	20 hrs at $12 per hour
Price per tile	$3	$4	$5	
Cost per tile	$1.50	$1.20	$1.6	
Profit per tile	$1.50	$2.8	$3.4	Maximize

$Maximize\ P = 1.5x + 2.8y + 3.4z$

$subjec\ to:\quad x + y + z \geq 300$

$5x + 2y + 2z \leq 1500$

$3y + 5z \leq 1200$

$All\ variables\ are\ non-negative.$

#4. The Great Canadian Coffee Company imports coffee beans and is attempting to create a coffee blend that will maximize profit and appeal to Canadian tastes. Two major characteristics of coffee are acidity and body. Consumer studies have indicated that Canadians prefer a coffee blend that is not overly acidic and that can be characterized as full-bodied. The company packages its coffee blends in 150 gram vacuum-sealed bags and sells 10,000 bags a day. Each bag produces $1.20 sales revenue. The four types of coffee bean that the company imports for blending have the following characteristics:

Coffee Bean	Brazilian	Colombian	Jamaican	Hawaiian
Cost per kilogram ($)	4.60	4.40	3.80	3.60
Acidity coefficient	7	2	6	3
Body coefficient	4	6	3	5
Daily availability (kg)	400	500	300	400

The company has decided that a Canadian blend should have an average acidity coefficient that is not more than 5, and the average body coefficient that is at least 4. Also, each 150 gram bag should contain at most 30% Jamaican coffee and at least 20% Colombian coffee. **Formulate an LP model for the problem. DO NOT SOLVE.**

Suppose B kg of Brazilian, C kg of Colombian, J kg of Jamaican and H kg of Hawaiian coffee bean will be used in the blend.

- If the daily demand is 10,000 bags, then the daily requirement for coffee bean would be 1500 kg (=$150\ g \times 10{,}000\ bags$).
- Because the selling price is $1.20 per bag and the daily demand is 10,000 bags, the revenue is a fixed number. Consequently, to maximize the profit is equivalent to minimize the blending cost.
- Please that the problem could also be formulated minimize the cost for one bag. In this case, the model structure remains the same, but parameters would be a little different.

$Minimize\ C = 4.6B + 4.4C + 3.8J + 3.6H$

$subject\ to:\quad B + C + J + H \geq 1500$ Total weight (kg)

$7B + 2C + 6J + 3H \leq 7500$ Acidity, no more than 5 units per kg

$4B + 3C + 6J + 5H \geq 6000$ Body coefficient, at least 4 units per kg

$J \leq 450$ $(= 0.3 \times 1500)$ Up limit on Jamaican (no more than 30%)

$C \geq 300$ $(= 0.2 \times 1500)$ Lower limit on Colombian (at least 20%)

Limit on supply	Non-negativity
$B \leq 400$	$B \geq 0$
$C \leq 500$	$C \geq 0$
$J \leq 300$	$J \geq 0$
$H \leq 400$	$H \geq 0$

Alternative solution:

$Minimize\ C = 4.6B + 4.4C + 3.8J + 3.6H$

$subject\ to$: $B + C + J + H \geq 1500$

$7B + 2C + 6J + 3H \leq 5(B + C + J + H)$ $\quad 2B - 3C + J - 2H \leq 0$

$4B + 3C + 6J + 5H \geq 4(B + C + J + H)$ $\quad -C + 2J + H \geq 0$

$J \leq 0.3(B + C + J + H)$ $\quad -0.3B - 0.3C + 0.7J - 0.3H \leq 0$

$C \geq 0.2(B + C + J + H)$ $\quad -0.2B + 0.8C - 0.2J - 0.2H \geq 0$

$B \leq 400$ $\quad B \geq 0$

$C \leq 500$ $\quad C \geq 0$

$J \leq 300$ $\quad J \geq 0$

$H \leq 400$ $\quad H \geq 0$

#5. Jorge daSilva, president of Hardrock Concrete Company, has plants in three locations, and is currently working on three major construction projects, located in different sites. The shipping costs per truckload of concrete, plant capacities and project requirements are given in the table below. Set up the transportation model for the company, which minimizes the total shipping cost. **Formulate an LP model for the problem. DO NOT SOLVE.**

	Shipping to			
From	Project A	Project B	Project C	Plant Capacity
Plant 1	\$10	\$4	\$11	70
Plant 2	\$12	\$5	\$8	50
Plant 3	\$9	\$7	\$6	30
Project Requirement	40	50	60	150

Let A1, B1 & C1 be number of truckloads of concrete shipped from Plant 1 to construction A, B & C, respectively. Similarly, define A2, B2, & C2; A3, B3 & C3.

Min $Cost = 10A1 + 4B1 + 11C1 + 12A2 + 5B2 + 8C2 + 9A3 + 7B3 + 6C3$

s.t. $A1 + B1 + C1 \leq 70$ $\quad A1 + A2 + A3 \geq 40$

$A2 + B2 + C2 \leq 50$ $\quad B1 + B2 + B3 \geq 50$

$A3 + B3 + C3 \leq 30$ $\quad C1 + C2 + C3 \geq 60$

All variables ≥ 0