SOLUTION KEY TO ASSIGNMENT #4

Due: 1 pm, Friday, October 29th 2010

#1. For question #3 in A3, the **Production of Tiles in Three Styles.**

a) Set the LP model in Excel; solve it using Solver; and print a one-page Excel output with the model, answer report and sensitivity report.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**A4-q1: Three styles of tiles.**								**Microsoft Excel 12.0 Sensitivity Report**							
2			Styles						**Worksheet: [1205-SA4-10F.xlsx]A4-#1**							
3		I	II	III					**Report Created: 10/25/2010 1:55:41 PM**							
4		140	400	0												
5	price per tile	3	4	5												
6	cost per tile	1.5	1.2	1.6					Adjustable Cells							
7	profit per unit	1.5	2.8	3.4	1330.00	Maximize						**Final**	**Reduce**	**Objectiv**	**Allowab**	**Allowable**
8					LHS		RHS	cost per hour		**Cell**	**Name**	**Value**	**Cost**	**Coefficie**	**Increase**	**Decrease**
9	Total tiles to make	1	1	1	540	>=	300			$B$4	I	140	0	1.5	3.25	1.5
10	Shaping time	5	2	2	1500	<=	1500	$18		$C$4	II	400	0	2.8	1E+30	0.52
11	Painting time	0	3	5	1200	<=	1200	$12		$D$4	III	0	-0.867	3.4	0.867	1E+30
12																
13	**Microsoft Excel 12.0 Answer Report**								Constraints							
14	**Worksheet: [1205-SA4-10F.xlsx]A4-#1**											**Final**	**Shadow**	**Constrai**	**Allowab**	**Allowable**
15	**Report Created: 10/25/2010 1:55:40 PM**									**Cell**	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
16	Target Cell (Max)									$E$9	Total tiles	540	0	300	240	1E+30
17		**Cell**	**Name**	**Origina**	**Final Value**					$E$10	Shaping	1500	0.3	1500	1E+30	700
18		$E$7	profit per	1330	1330					$E$11	Painting	1200	0.733	1200	1050	1200
19	Adjustable Cells															
20		**Cell**	**Name**	**Origina**	**Final Value**											
21		$B$4	I	140	140											
22		$C$4	II	400	400											
23		$D$4	III	0	0											
24	Constraints															
25		**Cell**	**Name**	**Cell Va**	**Formula**	**Status**	**Slack**									
26		$E$9	Total tile	540	$E$9>=$G	Not Bin	240									
27		$E$10	Shaping	1500	$E$10<=$	Binding	0									
28		$E$11	Painting	1200	$E$11<=$	Binding	0									
29																

b) In the optimal solution, how many tiles of each style will be produced? What is the maximum profit?

In the optimal solution, 140 tiles would be made in Style I, 400 in Style II, and no tile would be made in Style III.

The maximum total profit is $1,330.

c) In the optimal solution, how much shaping time will be used? How much painting time will be used?

Both shaping and painting time are fully used, that is, 1,500 hours of shaping time, and 1,200 hours of painting time are used according to the optimal solution.

d) In the LP model, there is a painting constraint, $3y + 5z \leq 1200$, where 1200 minutes (20 hours) are available to the production.

In each of the following cases, run Solver with a given painting time (all the other parameters in the model remain the same); write down the new profit (the value of the objective function); and find how much the profit has increased (or decreased), that is, $\Delta P = the\ new\ profit - original\ Profit\ found\ in\ (b)$.

Changes in painting time	ΔRHS	New Profit	ΔP	Profit increase per min
1) RHS = 1201	+1 min	1330.73	1330.73 − 1330= 0.73	$\frac{\Delta P}{\Delta RHS}$ = \$0.73 per min
2) RHS = 1260	+60 min	1374.00	1374.00 − 1330= 44.00	$\frac{\$44}{60\ min} = \$0.73\ per\ min$
3) RHS = 1199	−1 min	1329.27	1329.27 − 1330= −0.73	$\frac{-\$0.73}{-1} = \$0.73\ per\ min$
4) RHS = 1140	−60 min	1286.00	1286.00 − 1330= −44.00	$\frac{-\$44}{-60} = \$0.73\ per\ min$

If we acquire one extra minute, the profit will increase by $0.73.

#2. For question #4 in A3, **the Great Canadian Coffee Company**.

a)Set this model in Excel; solve it using Excel Solver and print a one-page Excel output with the model, answer report and sensitivity report.

	A	B	C	D	E	F	G	H	I
1	**The Great Canadian Coffe**	Brazilian	Colombian	Jamaican	Hawaiian				
2	# kg	300	500	300	400			Profit	
3	Cost per kg	4.6	4.4	3.8	3.6	6160	Minimize	5840	Maximum
4									
5	Total weight	1	1	1	1	1500	≥	1500	kg
6	Body coefficient	4	6	3	5	7100	≥	6000	≥4×1500kg
7	At least 20% of Colombian		1			500	≥	300	
8	Acidity coefficient	7	2	6	3	6100	≤	7500	≤5×1500kg
9	At most 30%of Jamaican			1		300	≤	450	
10	Up limit for Brazilian	1				300	≤	400	kg
11	Up limit for Colombian		1			500	≤	500	kg
12	Up limit for Jamaican			1		300	≤	300	kg
13	Up limit forHawaiian				1	400	≤	400	kg

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [1205-SA4-10F.xlsx]A4-#2
Report Created: 10/25/2010 7:40:42 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficien	Allowable Increase	Allowable Decrease
$B$2	# kg Brazil	300	0	4.6	1E+30	0.2
$C$2	# kg Colon	500	0	4.4	0.2	1E+30
$D$2	# kg Jama	300	0	3.8	0.8	1E+30
$E$2	# kg Hawa	400	0	3.6	1	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$6	Total weigl	1500	4.6	1500	100	275
$F$7	Body coeff	7100	0	6000	1100	1E+30
$F$8	At least 2(	500	0	300	200	1E+30
$F$9	Acidity coe	6100	0	7500	1E+30	1400
$F$10	At most 3(	300	0	450	1E+30	150
$F$11	Up limit fo	300	0	400	1E+30	100
$F$12	Up limit fo	500	-0.2	500	300	100
$F$13	Up limit fo	300	-0.8	300	150	100
$F$14	Up limit fo	400	-1	400	300	100

Microsoft Excel 12.0 Answer Report
Worksheet: [1205-SA4-10F.xlsx]A4-#2
Report Created: 10/25/2010 7:40:42 PM

Target Cell (Min)

Cell	Name	riginal Valı	Final Value
$F$4	Cost per kg	0	6160

Adjustable Cells

Cell	Name	riginal Valı	Final Value
$B$2	# kg Brazilian	0	300
$C$2	# kg Colombian	0	500
$D$2	# kg Jamaican	0	300
$E$2	# kg Hawaiian	0	400

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$6	Total weight	1500	$F$6>=$H$6	Binding	0
$F$7	Body coefficient	7100	$F$7>=$H$7	Not Binding	1100
$F$8	At least 20% of Cc	500	$F$8>=$H$8	Not Binding	200
$F$9	Acidity coefficient	6100	$F$9<=$H$9	Not Binding	1400
$F$10	At most 30%of Jar	300	$F$10<=H	Not Binding	150
$F$11	Up limit for Brazilia	300	$F$11<=H	Not Binding	100
$F$12	Up limit for Colomt	500	$F$12<=H	Binding	0
$F$13	Up limit for Jamaic	300	$F$13<=H	Binding	0
$F$14	Up limit forHawaiia	400	$F$14<=H	Binding	0

b) In the optimal solution, how many kilograms of each type of coffee bean will be used to make 10,000 bags of the Canadian blend? What is the maximum daily profit? How many grams of each type of coffee bean will be used in each 150 gram bag?

In the optimal solution, 300 kg of Brazilian, 500 kg of Colombian, 300 kg of Jamaican and 400 kg of Hawaiian coffee bean will be used to make the Canadian blend.

The maximum daily profit is $5,840. ($P = R - C = \$1.2 \times 10,000 - 6160.$)

Each bag will contain 30 grams of Brazilian, 50 grams of Colombian, 30 grams of Jamaican and 40 grams of Hawaiian coffee bean.

c) A daily supply is set for all the four types of coffee bean. In the optimal solution, are all the daily supplies used up? How many kilograms are left for each type?

With the optimal solution, the entire daily supplies of coffee bean are used up, except Brazilian coffee bean. For Brazilian coffee bean, of 400 kg of daily supply, 300 kg are used in the blend, and 100 kg left over.

#**3.** The Electrontech Corporation manufactures two industrial electrical devices: generators and alternators. Each product can be manufactured in one of the company's two plants. The following table shows the production time (in hours/unit) and cost (in $/unit) at each plant.

Each month, the company's management set a budget of $30,000 for the production of generators, and $10,000 for the production of alternators. Each month, 500 hours of production time are available in Plant I, and 600 hours are available in Plant II.

	Production Time (hours/unit)		Production Costs ($/unit)	
	Plant I	Plant II	Plant I	Plant II
Generator	4	3	120	100
Alternator	7	5	150	170

The company sells each generator at $350 and each alternator at $250. Due to market consideration, the company wants to produce at least one alternator for every three generators manufactured. The company has already received an order for 25 generators and 15 alternators.

How many units of each product should the company produce next month to maximize its profit? **Formulate an LP model for the problem. DO NOT SOLVE IT.**

Let G_1 = number of generators made in Plant I, and G_2 = number of generators made in Plant II;

A_1 = number of generators made in Plant I, and A_2 = number of generators made in Plant II;

Profit per unit:

	Plant I	**Plant II**
Generator	**350 – 120 = 230**	**350 – 100 = 250**
Alternator	**250 – 150 = 100**	**250 – 170 = 80**

$$Max\ P = 230G_1 + 250G_2 + 100A_1 + 80A_2$$

$s.t.$

$4G_1 + 7A_1 \le 500$	**(Production time in Plant I)**
$3G_2 + 5A_2 \le 600$	**(Production time in Plant II)**
$120G_1 + 100G_2 \le 30,000$	**(Budget limit on Generators)**
$150A_1 + 170A_2 \le 10,000$	**(Budget limit on Alternators)**
$G_1 + G_2 \ge 25$	**(Lower limit on Generators)**
$A_1 + A_2 \ge 15$	**(Lower limit on Alternators)**
$-G_1 - G_2 + 3A_1 + 3A_2 \ge 0$	**(Ratio relationship)**

$All\ variables\ are\ non-negative.$

#4. A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required each day is given in the table below. Union rules state that each full-time employee must work five consecutive days and then receives two days off. For example, an employee who works Monday to Friday must be off on Saturday and Sunday.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Min employees required	17	13	15	19	14	16	11

a) The post office wants to meet its daily requirements using only full-time employees. Its objective is to minimize the number of full-time employees that must be hired. Formulate the problem as an IP model.

Employees start on →	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Monday	M			R	F	S	SS
Tuesday	M	T			F	S	SS
Wednesday	M	T	W			S	SS
Thursday	M	T	W	R			SS
Friday	M	T	W	R	F		
Saturday		T	W	R	F	S	
Sunday			W	R	F	S	SS

Suppose M = number of employees start working on Monday, and similarly define T, W, R, F, S, and SS.

$$Min\ N = M + T + W + R + F + S + SS$$

s.t.

$$M + R + F + S + SS \geq 17$$

$$M + T + F + S + SS \geq 13$$

$$M + T + W + S + SS \geq 15$$

$$M + T + W + R + SS \geq 19$$

$$M + T + W + R + F \geq 14$$

$$T + W + R + F + S \geq 16$$

$$W + R + F + S + SS \geq 11$$

All variables are integer.

b) Implement your IP model in Excel.

	A	B	C	D	E	F	G	H	I	J	K
1	**#4. Post Office Staff Scheduling**										
2		Monday	Tuesday	Wednesda	Thursday	Friday	Saturday	Sunday	Total # hired		
3	# workers	9	2	3	4		7	1	26	Minimize	
4											
5	Monday	1			1	1	1	1	21	≥	17
6	Tuesday	1	1			1	1	1	19	≥	13
7	Wednesday	1	1	1			1	1	22	≥	15
8	Thursday	1	1	1	1			1	19	≥	19
9	Friday	1	1	1	1	1			18	≥	14
10	Saturday		1	1	1	1	1		16	≥	16
11	Sunday			1	1	1	1	1	15	≥	11

An initial solution is shown in cells B3:H3, and 26 people have to be hired to provide required services.

c) Run Solver to find an optimal solution. What is the optimal schedule? How many employees do you need to hire?

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total # hired	
# workers	7	4	0	8	0	4	0	23	Minimize

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total # hired	
# workers	6	6	0	7	0	4	0	23	Minimize

Note that there are multiple optimal schedules in this case, which results in the same minimum total hiring.

d) In the optimal solution found in part (c), how many employees start working on Saturday?

Four people will start working on Saturday.

In general, this group of employees will not be happy because they never have a weekend off. Is it possible to improve the optimal solution and make it "fairer"? That is to treat all employees in an equal fashion.

To treat everyone equally, we may rotate all the shifts according to the number of workers in each group. For examples, an employee in the first group will start working on Monday for 7 weeks, on Tuesday for 4 weeks, on Thursday for 8 weeks, and then Saturday for 4 weeks. Similarly for people in other groups. This way, it takes 23 weeks to make a cycle, and every employee will be treated exactly same.