Math 1341: Linear Algebra
Winter 2005

Time & Location: Tuesday 13:00-14:30 & Thursday 11:30 - 13:00 in SITE B0138.

Professor:

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Problem Sessions (DGD): Friday 13:00-14:30 in Colonel By Hall Room C03. The DGD is a part of the course and participation is mandatory.

Office Hours: TBA

Prerequisite: MAT 1340 or Ontario grade 12U Geometry and discrete mathematics or OAC Algebra and Geometry.

Course Website: Information about the course will be occasionally updated on the following website:

http://www.mathstat.uottawa.ca/~faridi/1341.html

Text: *Elementary Linear Algebra*, 2nd edition, by W. Keith Nicholson (McGraw-Hill Ryerson).

Course Overview: We will cover the topics stated in the official course description. In the textbook, these are approximately the following sections: 1.1–1.5, 2.1–2.5, 3.1–3.3, 3.5, 4.1–4.6, 4.9, 5.1–5.3. The material in 3.1–3.3 and 3.5 is part of the prerequisites and will be done in the beginning of the course. It will be examined in the first test on Friday, Jan. 28 (see below for more about this test). Also, note that we may not cover all the material of a section as it is given in the book. You will of course only be responsible for the material covered in class.

Tests: There will be three tests, all written during your DGD sessions, and scheduled as follows:

Test 1: Friday, Jan. 28
Test 2: Friday, Feb. 18
Test 3: Friday, April 1

There will be no makeup tests. You are allowed to miss a maximum of 1 test (this policy takes care of emergencies and conflicts of schedule). The grade of the final exam will substitute the grade of the missed test (or one lowest test grade, if that is to your advantage).

Your Final Grade: If your score on the final exam is less than 50% then your final grade is the grade of the final exam, i.e., F if the grade of the final exam is less than 40%, or E if the grade of the final exam lies between 40% and 50%.

If the grade of the final exam is at least 50%, then the final grade will be the weighted average of your scores on the three tests, each worth 15% and the final exam (55%). One test score may be replaced by the score of the final exam, if this is to your advantage.

Final Exam Deferral for Medical Reasons: See the website of the Faculty of Science: *www.science.uottawa.ca*

Calculators: Calculators are not required for this course. You will not be allowed to use calculators during any of the exams.

Important dates for the Fall 2004 semester: For details please see *www.science.uottawa.ca*

Extra Help: Besides office hours, you can use the following resources to succeed in this course.

- The **S.O.S. Math** drop-in center (*http://www.mathstat.uottawa.ca/centre/dropin.htm*) has two locations and is staffed by consultants willing to help (on a one-to-one basis) with all first year math courses. The schedule from January 10 to April 11 is the following:

CENTER 1 (MARION-021):	Monday-Thursday: 10:00-7:00	Friday: 10:00-4:00
CENTER 2 (KED-B03):	Monday-Thursday: 10:00-4:00	Friday: Closed

There will be a special schedule for the study week of Feb. 21-25.

- The **Linear Algebra test bank** is a collection of multiple choice questions. It can be accessed by every student registered in this course via the Center for Mediated Teaching and Learning at `"https://maestro.uottawa.ca/`
- If you want to know more about the **applications of linear algebra**, visit the web site "Linear Algebra close to Earth" at `http://aix1.uottawa.ca/~jkhoury/linearmain.htm`.

Diagnostic component of test 1 (Friday, Jan. 28): A major part of this test will diagnostic, i.e., material taken from the prerequisites of the course (high school vector geometry and complex numbers). This material will be reviewed in the first three classes and the first two problem sessions (Friday, Jan. 7 and 14). The following is a list of the relevant material for the diagnostic part of the test 1:

- find the equations of a line in 3-space, given sufficient data,
- find the equation of a plane in 3-space, given sufficient data,
- determine the intersection of 2 planes in 3-space,
- determine the intersection of 2 lines in 3-space,
- determine the intersection of a line and a plane in 3-space,
- use the cross product to compute (i) a normal to a plane and (ii) the area of a triangle in 2 or 3-space,
- use the dot product to compute (i) the angle between 2 vectors in 2 or 3-space, (ii) the length of a vector in 2 or 3-space, (iii) the projection of one vector on another, (iv) the distance from a point to a plane in 3-space, and (v) the distance from a point to a line in 2 or 3-space,
- add, subtract, multiply and divide complex numbers, and find the complex conjugate of a complex number, solve quadratic equations, with possibly complex solutions.

To practice, you should do the relevant suggested exercises (see the next page).

Suggested Exercises

Below is a list of suggested exercises that will help you prepare for the tests and the final exam. It is strongly recommended that you do all of them as we go along.

- *Section* 1.1: 4b, 6bd, 7b, 8b, 9b, 10b, 11b.
- *Section* 1.2: 1b, 3b, 4dfh, 6dfh, 8, 9bd, 14bdfhj.
- *Section* 1.3: 1bf, 4bd, 6, 7bdf, 8.
- *Section* 1.4: 1bdf, 2bdfg, 4, 7b, 8b, 10bdfhjl, 13b, 14, 16.
- *Section* 1.5: 2bdfh, 4, 5df, 8, 9b, 13bdfhj, 14, 16b, 19b, 22, 24.
- *Section* 2.1: 1d, 3fhjl, 6b, 8bdf, 10bdf, 12b, 13b, 16, 18b, 19, 20.
- *Section* 2.2: 4bdf, 6b, 8b, 10bdfh, 12b.
- *Section* 2.3: 2bdf, 4, 6b, 7bdfh, 8, 12b, 16b.
- *Section* 2.5: 1bdfhj, 2b, 3bd, 4, 6b, 7bdf, 8bdf, 9bd, 10b, 11bd, 12bd, 14, 16.
- *Section* 2.8: 1bd.
- *Section* 3.1: 2b, 3b, 4b, 6b, 7b, 8b, 10bd, 13b, 14b, 15b, 17bdfhj.
- *Section* 3.2: 1bf, 2b, 3bdfh, 4bf, 8bf, 9b, 10b, 12, 16bdf, 22b, 24b.
- *Section* 3.3: 2bdfhjl, 4b, 6, 8b, 10bdf, 12bd, 13bdfhj, 14b, 16b, 19b, 20b, 21b, 22, 23b, 25bdfh.
- *Section* 3.5: 1b, 3b, 4bd, 10, 12.
- *Section* 4.1: 2bd, 3bd, 4bd, 5bdf, 6bd, 7b, 8bdf, 10, 14, 15b, 16b, 18, 20, 22, 24b.
- *Section* 4.2: 1bd, 2bdf, 3b, 4bd, 5b, 7bdfh, 9bd, 14.
- *Section* 4.3: 1bd, 2bd, 3bd, 4bd, 6b, 7bd, 8b, 9bd, 10bdf, 16b, 20, 24.
- *Section* 4.4: 1b, 3b, 4b, 5b, 6b, 7bdf, 8, 10, 12, 16b.
- *Section* 4.5: 1bd, 2b, 3b, 5b, 6b, 7bd, 9bdfh, 10, 16.
- *Section* 4.6: 1b, 2b, 3b, 9bdf, 10, 12, 14b.
- *Section* 4.9: 1bdf, 2, 4, 6b, 10, 14b, 18b, 20.
- *Section* 5.1: 1bdf, 2bd, 3bd, 6bdf, 7bdf, 8bdf, 9bdf, 10b, 12b, 18, 20b, 24b, 26, 28.
- *Section* 5.2: 1bdfhjl, 2bdfh, 3bdfhjl, 4b, 5b, 10, 12b, 18, 20b, 22, 24bd.
- *Section* 5.3: 1bdfhjl, 2bdfh, 3bdfh, 4b, 5b, 6, 10b, 12, 14b, 16, 22.

The Invertible Matrix Theorem:
For easier references, below is a theorem that we will establish during the course in several easy steps. You will need to know parts of it for tests 2 and 3, and all of it for the final exam.

Let A be an $n \times n$-matrix. Then the following conditions are equivalent:

1. A is invertible, i.e., there exists a $n \times n$-matrix B such that $AB = I_n = BA$, where I_n is the $n \times n$-identity matrix.
2. There exists an $n \times n$-matrix C such that $CA = I_n$.
3. There exists an $n \times n$-matrix D such that $AD = I_n$.
4. The reduced row echelon form of A is I_n.
5. A has rank n.
6. The linear system $AX = b$ has a unique solution for every column b.
7. The linear system $AX = b$ has at least one solution for every column b.
8. The homogeneous linear system $AX = 0$ has only the trivial solution, i.e., $\operatorname{null} A = \{0\}$ where $\operatorname{null} A$ is the null space of A.
9. A^T is invertible.
10. $\det(A) \neq 0$.
11. 0 is not an eigenvalue of A.
12. The columns of A are linearly independent.
13. The columns of A are a basis of $\mathbb{R}^n$.
14. The columns of A span $\mathbb{R}^n$, i.e., $\operatorname{col} A = \mathbb{R}^n$.
15. The rows of A are linearly independent.
16. The rows of A are a basis of $\mathbb{R}^n$.
17. The rows of A span $\mathbb{R}^n$, i.e., $\operatorname{row} A = \mathbb{R}^n$.
18. The linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$, given by $X \mapsto AX$, is invertible.

Remarks.

(a) Note the equivalence of (1) and (2) says that if $AB = I_n$ holds for square matrices then we also have $BA = I_n$, and similarly for (3). Hence, to check invertibility it is sufficient to check one of the two equations, $AB = I_n$ or $BA = I_n$.

(b) Note the difference between (6) and (7)! Here (6) is the stronger assertion, while (7) says that as soon as we have a solution of $AX = b$ for **all** b we then immediately know, by the theorem, that the solution is unique.

(c) The equivalence of (8) and (11) follows right from the definitions: 0 is an eigenvalue of A if and only if the homogeneous linear system $AX = 0$ has a non-trivial solution.

(d) A map $f : X \to Y$ from a set X to another set Y is *invertible* if there exists a map $g : Y \to X$ such that $f \circ g$ is the identity map on Y, i.e., $f(g(y)) = y$ for all $y \in Y$, and $g \circ f$ is the identity map on X, i.e., $g(f(x)) = x$ for all $x \in X$. Equivalently, f is *surjective* (sometimes also called "onto") and *injective* (sometimes also called "one-to-one"). In this case, the map g is unique and denoted $g = f^{-1}$.

(e) If A is invertible, then not only is T_A invertible but also its inverse is a linear transformation, namely $(T_A)^{-1} = T_{A^{-1}}$.