7.2 30, 42, 52. 7.5 * 20, 24, 58 Total 40'

7.3 * 38

7.4 * 34, 42, 50, 52

7.2 30. $(-2,1,0)$, $2x+5y-z=20$ 3'

$$D=\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$

$$=\frac{|2(-2)+5(1)+(-1)0-20|}{\sqrt{2^2+5^2+(-1)^2}}=\frac{19}{\sqrt{30}}=\frac{19\sqrt{30}}{30}\approx 3.469$$

42. $y^2+z^2-x^2=1$

Trace in xy-plane $(z=0)$: $y^2-x^2=1$ hyperbola 1' 3'

Trace in xz-plane $(y=0)$: $z^2-x^2=1$ hyperbola 1'

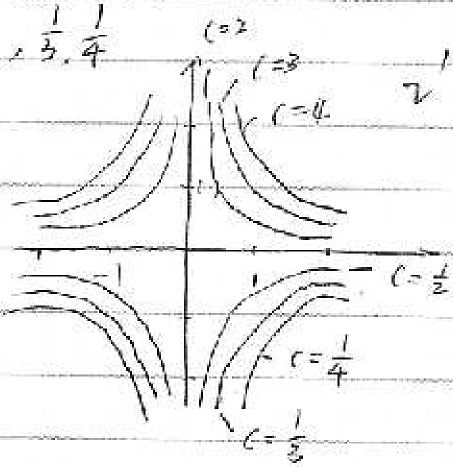
Trace in yz-plane $(x=0)$: $y^2+z^2=1$ Circle 1'

52. $x^2+\frac{y}{4}-z^2=0$ The graph is an elliptic cone 3'

7.3 38. $z=e^{xy}$ $c=1,2,3,4,\frac{1}{2},\frac{1}{3},\frac{1}{4}$

$c=1$	$1=e^{xy}$	$0=xy$
$c=2$	$2=e^{xy}$	$\ln 2=xy$
$c=3$	$3=e^{xy}$	$\ln 3=xy$
$c=4$	$4=e^{xy}$	$\ln 4=xy$
$c=\frac{1}{2}$	$\frac{1}{2}=e^{xy}$	$-\ln 2=xy$
$c=\frac{1}{3}$	$\frac{1}{3}=e^{xy}$	$-\ln 3=xy$
$c=\frac{1}{4}$	$\frac{1}{4}=e^{xy}$	$-\ln 4=xy$

3'



2'

5'

The level curves are hyperbolas

7.4 34. $w=\frac{xy}{x+y+z}$ $(1,2,0)$

$w_x=\frac{y(y+z)}{(x+y+z)^2}$ at $(1,2,0)$ $w_x=\frac{4}{9}$ 1' 3'

$w_y=\frac{x(x+z)}{(x+y+z)^2}$ at $(1,2,0)$ $w_y=\frac{1}{9}$ 1'

$w_z=\frac{-xy}{(x+y+z)^2}$ at $(1,2,0)$ $w_z=-\frac{2}{9}$ 1'

42. $f(x,y)=\ln(x^2+y^2+1)$

$f_x(x,y)=\dfrac{2x}{x^2+y^2+1}$ 1' $=0 \Longrightarrow x=0$

$f_y(x,y)=\dfrac{2y}{x^2+y^2+1}$ 1' $=0 \longrightarrow y=0$ (3')

solution: $(0,0)$. 1'

6. $z=x^2-y^2$ $(-2,1,3)$

a) $\dfrac{\partial z}{\partial x}=2x$ 1' at $(-2,1,3)$ $\dfrac{\partial z}{\partial x}=-4$ 1' (3')

b) $\dfrac{\partial z}{\partial y}=-2y$ 1' at $(-2,1,3)$ $\dfrac{\partial z}{\partial y}=-2$.

12. $z=x^4-3x^2y^2+y^4$

$\dfrac{\partial z}{\partial x}=4x^3-6xy^2$ $\dfrac{\partial^2 z}{\partial y\partial x}=-12xy$ ✓ (3')

$\dfrac{\partial z}{\partial y}=-6x^2y+4y^3$ $\dfrac{\partial^2 z}{\partial x\partial y}=-12xy$ ✓

7.5 20. $f(x,y)=-\dfrac{4x}{x^2+y^2+1}$

The first partial derivatives of f.

$f_x(x,y)=\dfrac{4(x^2-y^2-1)}{(x^2+y^2+1)^2}$ 1' and $f_y(x,y)=\dfrac{8xy}{(x^2+y^2+1)^2}$ 1'

are zero at $(1,0)$ and $(-1,0)$. 1' (8')

$f_{xx}(x,y)=\dfrac{8x(-x^2+3y^2+3)}{(x^2+y^2+1)^3}$ 1'

$f_{yy}(x,y)=\dfrac{8x(x^2-3y^2+1)}{(x^2+y^2+1)^3}$ 1'

27
13

$$f_{xy}(x,y) = \frac{8y(-3x^2+y^2+1)}{(x^2+y^2+1)^3} \quad 1'$$

it follows that

$f_{xx}(1,0) = 2 > 0$ $\quad f_{xx}(-1,0) = -2 < 0$ $\quad$ and $f_{xx}(\pm1,0)f_{yy}(\pm1,0) - [f_{xy}(\pm1,0)]$

$= 4 > 0$

$1'$

Thus $(1,0,-2)$ is a relative minimum and $(-1,0,2)$ is a relative maximum. $1'$

24. $f_{xx}(x_0,y_0) = 20$ $\quad f_{yy}(x_0,y_0) = 8$ $\quad f_{xy}(x_0,y_0) = 9$

Since $f_{xx}(x_0,y_0) > 0$ $\quad 1'$

and $d = f_{xx}(x_0,y_0)f_{yy}(x_0,y_0) - [f_{xy}(x_0,y_0)]^2 = (20)(8) - (9)^2 > 0$ $\quad 1'$

$f(x_0,y_0)$ is a relative minimum. $1'$

$3'$

38. $R = 500p_1 + 800p_2 + 1.5p_1p_2 - 1.5p_1^2 - p_2^2$

The first partial derivatives of R are

$R_{p_1} = 500 + 1.5p_2 - 3p_1$

$R_{p_2} = 800 + 1.5p_1 - 2p_2$ $\quad 1'$

$3'$

Setting these equal to zero produces the system

$3p_1 - 1.5p_2 = 500$

$-1.5p_1 + 2p_2 = 800$

which yields $p_1 \approx \$586.67$ and $p_2 \approx \$840.00$ $\quad 1'$

The second Derivative Test verifies that these values maximize the total revenue. $1'$