Math 2341, Homework No. 2

Due: Thursday October 14th, 2004 in class

1. Let V be the subspace of $\mathbf{F}([0,1],\mathbf{R})$ generated by the functions f_1, f_2 and f_3 given by

$$f_1(x) = \frac{1}{x+1}, \quad f_2(x) = 2 - x \text{ and } f_3(x) = x^2$$

for all $x \in [0,1]$. Find a basis of the subspace U of V that consists of all the functions g of V such that $g(0) = g(1)$.

2. Let W be the solution set in $\mathbf{C}^3$ of the linear equation $ix + (1-i)y - z = 0$. Among the sets below, determine those that make a basis for W and *justify* your answer.

(i) $\{(i, 1+i, -1)\}$ (ii) $\{(1,1,1),(i,1,-i)\}$ (iii) $\{(0, 1-i, 1+i),(0,1,1-2i)\}$

3. Let $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 2 & 0 \\ -1 & 2 & 1 & 1 \end{pmatrix}$.

(i) Determine the rank of A and a basis for the set of solutions in $\mathbf{R}^4$ of the homogeneous system $AX = 0$.

(ii) Give a basis of the row space $\mathrm{Row}_{\mathbf{R}}(\mathrm{A})$ of A and a basis for the column space $\mathrm{Col}_{\mathbf{R}}(\mathrm{A})$ of A.

4. Let W_1 and W_2 be the subspaces of $\mathbf{M}_{2,2}(\mathbf{R})$ given by

$$W_1 = \left\{ \begin{pmatrix} x & -x \\ y & z \end{pmatrix} ; x,y,z \in \mathbf{R} \right\} \text{ and } W_2 = \left\{ \begin{pmatrix} a & b \\ -a & c \end{pmatrix} ; a,b,c \in \mathbf{R} \right\}.$$

Determine bases for W_1, W_2, $W_1 \cap W_2$ and $W_1 + W_2$, and verify the following equation

$$\dim(W_1 \cap W_2) + \dim(W_1 + W_2) = \dim W_1 + \dim W_2.$$

5. In each case, determine if the given function is a linear map.

(i) $T\colon \mathbf{R}^2 \to \mathbf{R}^3$ such that $T(x,y) = (xy, x, y)$.

(ii) $T\colon \mathbf{P}_2(\mathbf{R}) \to \mathbf{P}_3(\mathbf{R})$ such that $T(p(x)) = xp(x+1)$.

6. Let $T\colon \mathbf{M}_{2,2}(\mathbf{R}) \to \mathbf{M}_{2,2}(\mathbf{R})$ be the linear map given by $T(A) = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} A$ for all matrices $A \in \mathbf{M}_{2,2}(\mathbf{R})$. Determine a basis for the image and a basis for the kernel of T.