assignment # 5.

7.6 # 34. 48 ; 7.8 # 24, 30, 44 ; 7.9 # 12, 22, 36

7.6 34. $F(x,y,z,\lambda) = (x-4)^2 + y^2 + z^2 - \lambda(x^2+y^2-z^2)$ 1'

$F_x = 2(x-4) - 2x\lambda = 0, \quad 2x(1-\lambda) = 8$

$F_y = 2y - 2y\lambda = 0 \quad 2y(1-\lambda) = 0$ 2'

$F_z = 2z + 2z\lambda = 0 \quad 2z(1+\lambda) = 0$

$F_\lambda = -x^2 - y^2 + z^2 = 0, \quad z = \sqrt{x^2+y^2}$

4'

From F_y, we have $y=0$ or $\lambda = 1$, from F_x, we know that $\lambda \neq 1$ (since $0 \neq 8$) thus, $y=0$. From F_λ and F_z, we now have $x = z$ and $\lambda = -1$. 1'

Thus $x=2$ $y=0$ and $z=2$ and we have $d = \sqrt{(2-4)^2+0^2+2^2} = 2\sqrt{2}$

48. Let $f(x,y) = xy$ be the area function 1'

Constraint : $g(x,y) = 2x + 2y - P$ 1'

$F(x,y,\lambda) = xy - \lambda(2x+2y-P)$

$F_x = y - 2\lambda = 0$

$F_y = x - 2\lambda = 0$ 1'

$F_\lambda = -2x - 2y + P = 0$

4'

Solving this system, $y = 2\lambda$, $x = 2\lambda$ and

$-2x - 2y + P = 0$

$-2(2\lambda) - 2(2\lambda) + P = 0$

$P = 8\lambda$

$\lambda = \frac{P}{8}$ 1'

Thus $x = y = \frac{P}{4}$ (square!)

and Area $= f\left(\frac{P}{4}, \frac{P}{4}\right) = \frac{P^2}{16}$

7.8 24. Since (for fixed y) $\lim\limits_{b\to\infty}\left[-\frac{1}{2}ye^{-(x^2+y^2)}\right]_0^b = \frac{1}{2}ye^{-y^2}$ 2'

we have the following

$$\int_0^\infty\int_0^\infty xye^{-(x^2+y^2)}\,dx\,dy = \int_0^\infty \frac{1}{2}ye^{-y^2}\,dy = \lim_{b\to\infty}\left[-\frac{1}{4}e^{-y^2}\right]_0^b = \frac{1}{4}$$ 2'

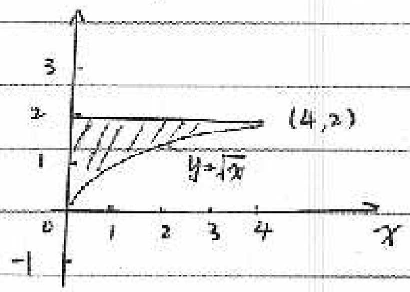
(4')

30. $\int_0^4\int_{\sqrt{x}}^2 dy\,dx = \int_0^4 (2-\sqrt{x})\,dx = \left(2x-\frac{2}{3}x^{\frac{3}{2}}\right)\Big|_0^4 = \frac{8}{3}$ 1'

$\int_0^2\int_0^{y^2} dx\,dy = \int_0^2 y^2\,dy = \frac{y^3}{3}\Big|_0^2 = \frac{8}{3}$ 1'

1'

(4')



1'

44. $xy=9$, $y=x$, $y=0$, $x=9$

The point of intersection of two graphs is given by equating $y=9/x$ and $y=x$, which yields $x=y=3$.

$A=\int_0^3\int_0^x dy\,dx + \int_3^9\int_0^{\frac{9}{x}} dy\,dx$ 1'

$=\int_0^3 x\,dx + \int_3^9 \frac{9}{x}\,dx$

$=\left[\frac{x^2}{2}\right]_0^3 + \left[9\ln|x|\right]_3^9$ 1' 1'

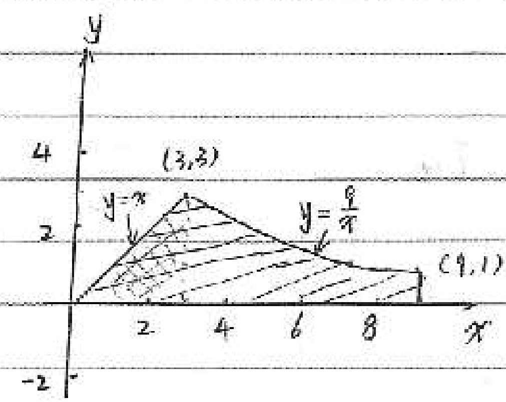
$=\frac{9}{2} + 9(\ln 9 - \ln 3)$

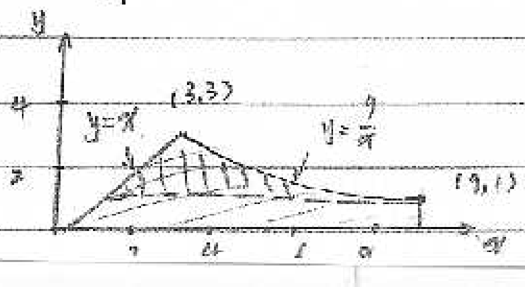
$=\frac{9}{2} + 9\ln 3$

$=\frac{9}{2}(1+2\ln 3)$ or

$A=\int_1^3\int_y^{\frac{9}{y}} dx\,dy + \int_0^1\int_y^9 dx\,dy$

(4')





$1'$ $1'$

7.4 12. $\int_0^4\int_0^{\sqrt{x}} \frac{y}{1+x^2}\,dy\,dx = \int_0^2\int_{y^2}^4 \frac{y}{1+x^2}\,dx\,dy$

set up the integral for **both** orders of integration

$(4')$

$$\int_0^4\int_0^{\sqrt{x}} \frac{y}{1+x^2}\,dy\,dx = \int_0^4 \left[\frac{y^2}{2(1+x^2)}\right]_0^{\sqrt{x}} dx \quad 1'$$

$$= \int_0^4 \frac{x}{2(1+x^2)}\,dx \quad 1'$$

$$= \frac{1}{4}\ln(1+x^2)\Big|_0^4$$

$$= \frac{1}{4}\ln 17 \approx 0.708 \quad 1'$$

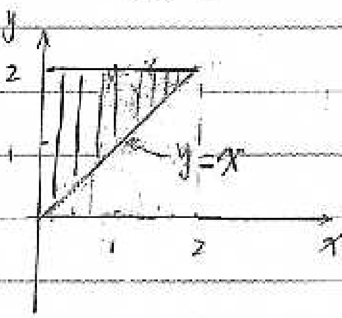
22. $V = \int_0^2\int_0^y (4-y^2)\,dx\,dy$ $1'$

or $V = \int_0^2\int_x^2 (4-y^2)\,dy\,dx$

$$= \int_0^2 y(4-y^2)\,dy \quad 1'$$

$$= \left[2y^2 - \frac{y^4}{4}\right]_0^2$$

$$= 4 \quad 1'$$



$(3')$

36. Average $= 2\int_0^1\int_0^y e^{x+y}\,dx\,dy$ $1'$

or Average $= 2\int_0^1\int_x^1 e^{x+y}\,dy\,dx$

$$= 2\int_0^1 \left[e^{x+y}\right]_0^y dy \quad 1'$$

$(3')$

$$= 2\int_0^1 (e^{2y}-e^y)\,dy$$

$$= 2\left[\frac{1}{2}e^{2y} - e^y\right]_0^1$$

$$= \left[e^{2y} - 2e^y\right]_0^1 \quad 1'$$

$$= e^2 - 2e + 1$$

$$= (e-1)^2$$