assignment #4

10.1 # 14, 44, 58; 10.2 # 8, 34, 52; 10.3 # 12, 18, 46

10.1 #14. This sequence converges since 1'

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = \sqrt{1} = 1 \quad 2'$$

(3')

#44. $a_n = \frac{x^{n+1}}{(n-1)!}$

(3')

#58. One example is $a_n = 100 - \frac{1}{n}$

(2')

10.2 #8. This series diverges by the nth-Term Test 1'

since $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$ 2'

(3')

#34. This series diverges by the nth-Term Test since 1'

$$\lim_{n\to\infty} \frac{n+1}{2n-1} = \frac{1}{2} \neq 0 \quad 2'$$

(3')

#52. $A_1 = \left(\frac{1}{4}\right) A$

$A_2 = \left(\frac{1}{4}\right) A_1 = \left(\frac{1}{4}\right)^2 A$

$A_3 = \left(\frac{1}{4}\right) A_2 = \left(\frac{1}{4}\right)^3 A$

$\vdots$ 1'

$A_n = \left(\frac{1}{4}\right)^n A$

(3')

The shaded area is given by $\sum_{n=1}^{\infty} A\left(\frac{1}{4}\right)^n = \sum_{n=0}^{\infty} A\left(\frac{1}{4}\right)^n - A = \frac{A}{1-1/4} - A$

$= \frac{4}{3}A - A = \frac{A}{3}$ 2'

which is $\frac{1}{3}$ the area of the square.

10.3 *12. This series converges since $p=\pi>1$ (2)

*18. Since $a_n = n\left(\frac{2}{3}\right)^n$, we have

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)\,2^{n+1}}{3^{n+1}}\cdot\frac{3^n}{n\cdot 2^n}\right|$$

$$= \lim_{n\to\infty}\frac{2(n+1)}{3n}$$

$$= \frac{2}{3} < 1$$

and the series converges. (3)

*46. This series diverges by the nth-Term Test since $\lim_{n\to\infty} \ln n = \infty$ (3)