

MATH 2341, HOMEWORK NO. 4

Due: Thursday November 25, 2004 in class

1. Let $\mathcal{E} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ be the standard basis of \mathbf{R}^3 and let $\mathcal{B} = \{(1, 0, 1), (1, 1, 0), (1, 1, 1)\}$ be another basis of \mathbf{R}^3 . Determine the matrices P and Q such that $[\mathbf{v}]_{\mathcal{B}} = P[\mathbf{v}]_{\mathcal{E}}$ and $[\mathbf{v}]_{\mathcal{E}} = Q[\mathbf{v}]_{\mathcal{B}}$ $\mathbf{v} \in \mathbf{R}^3$.

2. Let \mathcal{E} and \mathcal{B} be bases for \mathbf{C}^2 given by $\mathcal{E} = \{(1, 0), (0, 1)\}$ and $\mathcal{B} = \{(1, i), (-i, 2)\}$. Suppose $T: \mathbf{C}^2 \rightarrow \mathbf{C}^2$ is a linear transformation such that $[T]_{\mathcal{B}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Find $[T]_{\mathcal{E}}$, and find a formula for $T(x, y)$.

3. We consider the linear transformation $S: \mathbf{P}_1(\mathbf{R}) \rightarrow \mathbf{P}_2(\mathbf{R})$ given by $S(p(x)) = xp(x)$ and the linear transformation $T: \mathbf{P}_2(\mathbf{R}) \rightarrow \mathbf{P}_1(\mathbf{R})$ given by $T(p(x)) = p'(x)$.

(i) Give a formula for $T \circ S$.

(ii) Determine the matrices of T , S et $T \circ S$ relative to the bases $\mathcal{A} = \{1, x\}$ of $\mathbf{P}_1(\mathbf{R})$ and $\mathcal{B} = \{1, x, x^2\}$ of $\mathbf{P}_2(\mathbf{R})$.

(iii) Verify that $[T \circ S]_{\mathcal{A}} = [T]_{\mathcal{B}, \mathcal{A}}[S]_{\mathcal{A}, \mathcal{B}}$.

4. Let $T: \mathbf{M}_{2,2}(\mathbf{R}) \rightarrow \mathbf{M}_{2,2}(\mathbf{R})$ the linear transformation given by $T(A) = A \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ for all matrices $A \in \mathbf{M}_{2,2}(\mathbf{R})$.

(i) Determine the matrix $[T]_{\mathcal{B}}$ of T relative to the basis \mathcal{B} of $\mathbf{M}_{2,2}(\mathbf{R})$ given by

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

(ii) If T is invertible, calculate $[T^{-1}]_{\mathcal{B}}$ (*Hint: for a quick solution, use Theorem 6.12*).

5. Let $S: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be reflection across the plane defined by $x + y - z = 0$. Find a basis \mathcal{B} of \mathbf{R}^3 consisting of eigenvectors of S , and find $[S]_{\mathcal{B}}$.

6. In each case, determine the eigenvalues of the given matrix A , and for each of these eigenvalues give a basis for the corresponding eigenspace. If A is diagonalizable, find an invertible matrix P such that $P^{-1}AP$ is diagonal, and find $P^{-1}AP$.

(i) $A = \begin{pmatrix} -2 & 0 & 5 \\ 0 & 5 & 0 \\ -4 & 0 & 7 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$