Math 2341, Homework No. 5

Due: Monday December 6, 2004 in class

1. Let $\mathcal{B} = \{(2,2,-1),(2,-1,2),(-1,2,2)\}$.
   (i) Show that $\mathcal{B}$ is an orthogonal basis of $\mathbf{R}^3$ under the dot product.
   (ii) Determine $[(x,y,z)]_{\mathcal{B}}$ for a given point (x,y,z) of $\mathbf{R}^3$.
   (iii) Let $P\colon \mathbf{R}^3 \to \mathbf{R}^3$ be projection onto the plane U spanned by the vectors $(2,-1,2),(-1,2,2)$. Determine the matrix $[P]_{\mathcal{B}}$ of P with respect to the basis $\mathcal{B}$.
   (iv) Find the orthogonal projection of a given (x,y,z) onto U.

2. Let V be the real vector space of polynomial functions $f\colon [0,\infty[\to \mathbf{R}$. For all pairs of functions f and g of V, we set

$$\langle f,g\rangle = \int_0^\infty e^{-x} f(x)g(x)dx.$$

   (i) Show that this defines an inner product on V.
   (ii) Let U be the subspace of V generated by the functions 1 and x. Determine an orthogonal basis of U.
   (iii) Determine the orthogonal projection of the function x^2 onto U.
   (iv) Deduce an orthogonal bases for the subspace of V generated by the functions 1, x and x^2.

3. Consider $\mathbf{M}_{2,2}(\mathbf{R})$ with inner product $\langle\, A,B\rangle = \mathrm{tr}(B^tA)$. Let $U = \{A \in \mathbf{M}_{2,2}(\mathbf{R})\,;\, A = A^t\}$ be the subspace of 2×2 symmetric matrices. Find a basis for U and a basis for $U^\perp$.