MAT 1341B: Introduction to Linear Algebra

Instructor: Erhard Neher

Mini test 2 (November 19, 2004)

Family Name: ______________________________

First Name: ______________________________

Student number: ___________________________

Please read these instructions carefully:

- This is a closed book exam, and no notes of any kind are allowed. **The use of calculators, cell phones, pagers or any text storage or communication device is not permitted, and will be considered as fraud.**
- The exam has 5 multiple choice questions and 3 questions requiring detailed answers. The multiple choice questions are each worth 3 points each and no part marks will be given. **Please record your answers on the line "My answer: ____________" provided for each multiple choice question.**
- The last three questions are long-answer questions, and partial marks may be earned. Please be careful to include all details, and explain what you are doing. Question 7 depends on your student number. You must use the correct value to receive credit for the question.
- Read each question carefully - you will save yourself time and unnecessary grief later on. Where it is possible to check your work, do so.
- If you do not have enough space, use the back of the pages and clearly indicate this. The exam has 9 pages. You have 80 minutes to complete this exam.

Good luck!

Quest.	total MC	6.	7.	8.	Total
maximal	15	6	6	3	30
answer					

1. Which of the following are subspaces?

$$U = \left\{ \begin{bmatrix} -s & t & 2 & 0 \end{bmatrix}^T \in \mathbb{R}^4 \mid s, t \in \mathbb{R} \right\}$$
$$V = \left\{ \begin{bmatrix} s & t & 2s & t \end{bmatrix}^T \in \mathbb{R}^4 \mid s, t \in \mathbb{R} \right\}$$
$$W = \left\{ \begin{bmatrix} r & s & t \end{bmatrix}^T \in \mathbb{R}^3 \mid 2r - 5s + 7t = 0 \right\}$$

A. V only.
B. U only.
C. W only.
D. V and W only.
E. U and W only.
F. U and V only.
G. All three of them.

My answer:____________

2. (3 points) The eigenvalues of the matrix

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

are

A. $1 \pm \sqrt{2}\, i$.
B. $2 \pm \sqrt{3}$.
C. $2 \pm i$.
D. $3 \pm 4i$
E. $2 \pm \sqrt{2}\, i$
F. $1 \pm i$

My answer:____________

3. (3 points) Which of the following assertions are true for vectors $X_i \in \mathbb{R}^n$?

(i) If $3X_1 + 2X_2 - 5X_3 = 0$ then X_1, X_2, X_3 are linearly independent.

(ii) If none of the X_i are zero, then X_1, X_2, X_3 are a basis of *span* $\{X_1, X_2, X_3\}$.

(iii) If X_1, X_2, X_3 and X_4 are a basis of a subspace of $\mathbb{R}^n$ then $n \geq 4$.

(iv) Every nonzero subspace of $\mathbb{R}^n$ has a basis.

(v) If a subspace contains a spanning set of m vectors and k linearly independent vectors, then $k > m$.

A. (i) and (ii) only.
B. (i) and (iv) only.
C. (ii), (iii) and (v).
D. (ii) and (v) only.
E. (iii), (iv) and (v).
F. (iii) and (iv) only.

My answer:___________

4. (3 points) Find the projection $proj_U(X)$ for

$$U = span\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right\} \quad \text{and} \quad X = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}.$$

A. $\frac{1}{2}\,[2 \quad -1 \quad 2 \quad -1]^T$.

B. $\frac{1}{2}\,[2 \quad 1 \quad 2 \quad 1]^T$.

C. $\frac{1}{2}\,[1 \quad 3 \quad 1 \quad 3]^T$.

D. $\frac{1}{2}\,[2 \quad 3 \quad 2 \quad 3]^T$.

E. $\frac{1}{2}\,[-1 \quad 3 \quad -1 \quad 3]^T$.

F. $\frac{1}{2}\,[2 \quad 0 \quad 2 \quad 0]^T$.

My answer:___________

5. (3 points) The general solution of the system of linear differential equations

$$\begin{aligned} f_1' &= -f_1 + f_2 \\ f_2' &= 4f_1 + 2f_2 \end{aligned}$$

is of the form below, where c, $d \in \mathbb{R}$ are arbitrary:

A. $ce^{3x}\begin{bmatrix} 2 \\ -1 \end{bmatrix} + de^{2x}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

B. $ce^{3x}\begin{bmatrix} 1 \\ 4 \end{bmatrix} + de^{-2x}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

C. $ce^{3x}\begin{bmatrix} -4 \\ 1 \end{bmatrix} + de^{-2x}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

D. $ce^{3x}\begin{bmatrix} 2 \\ -1 \end{bmatrix} + de^{-2x}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

E. $ce^{-3x}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + de^{-2x}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

F. $ce^{-3x}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + de^{2x}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

My answer:____________

6. (6 points) Find a basis of the subspace

$$U = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \mid 3a - b + c + d = 0 \right\}$$

of $\mathbb{R}^4$. Justify your answer, i.e., you must either show that you have a basis or quote some theorems from class.

7. (6 points) In the matrix A below replace α with the **last** digit of your student number, write the new matrix obtained in this way next to the given A:

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 3 & 2 & 8 & 9 \\ 2 & 3 & 7 & \alpha \\ -1 & 3 & 1 & -3 \end{bmatrix} \quad = \quad \begin{bmatrix} 1 & -2 & 0 & 3 \\ 3 & 2 & 8 & 9 \\ 2 & 3 & 7 & \ldots \\ -1 & 3 & 1 & -3 \end{bmatrix}.$$

Find

(i) a basis for the row space of A,
(ii) a basis for the column space of A, and
(iii) the rank of A.

8. (3 points) Prove that a set $\{X_1, \ldots, X_m\}$ of orthogonal vectors in $\mathbb{R}^n$ is linearly independent.