

April 28, 2004

Professeur: A. Sebbar

## Mathematical Methods III –MAT 1303 FINAL EXAM

---

*This is a closed book exam. Non-graphing, standard scientific calculators are permitted*

---

**Problem 1 [3pts]:** Find the tangent line to the graph of the function  $f(x) = 2x^3$  at  $x = 1$ .

---

**Problem 2 [5pts]:**

- (1) Find  $y'$  if  $x^3 + y^3 = 6xy$ .
  - (2) Find the tangent line of  $x^3 + y^3 = 6xy$  at the point  $(3, 3)$ .
  - (3) At what points on the curve is the tangent line horizontal or vertical?
- 

**Problem 3 [3pts]:** Find

$$\int \frac{dx}{x^2 + 6x + 8}.$$

---

**Problem 4 [10pts]:** Determine whether or not each of the following improper integrals converge. If it does, evaluate it.

$$(a) : \int_1^{\infty} \frac{dx}{x^{\frac{3}{2}}}. \quad (b) : \int_0^1 \frac{dx}{x^{\frac{2}{3}}}. \quad (c) : \int_1^{\infty} \frac{dx}{x^{\frac{2}{3}}}. \quad (d) : \int_0^{\sqrt{2}} \frac{dx}{\sqrt{2-x}}. \quad (e) : \int_0^{\infty} e^{-2x} dx.$$

---

**Problem 5 [3pts]:** Find the centre and the radius of the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 4 = 0.$$


---

**Problem 6 [7pts]:** Let  $f(x, y) = x^4 + y^4 - 4xy + 1$ .

- (1) Find the partial derivatives  $f_x$  and  $f_y$ .
  - (2) Determine the critical points of  $f(x, y)$ .
  - (3) For each critical point, determine whether it is a local minimum, local maximum or a saddle point.
- 

**Problem 7 [3pts]:** Evaluate the double integral

$$\int_0^1 \int_y^{2y} (1 + 2x^2 + 2y^2) dx dy$$


---

**Problem 8 [3pts]:** Use a double integral to find the area of the region bounded by the graphs of the equation:

$$xy = 9, y = x, y = 0, x = 9.$$


---

**Problem 9 [2pts]:** Find the  $n$ th term of the sequence:  $\frac{1}{3}, -\frac{2}{9}, \frac{4}{27}, -\frac{8}{81}, \dots$

---

**Problem 10 [8pts]:** Determine the convergence or divergence of the following sequences. If a sequence converges find its limit:

$$(a) : a_n = \frac{2^n}{3^n}, \quad (b) : a_n = \frac{3^n}{2^n}, \quad (c) : a_n = \frac{n!}{(n+1)!},$$

$$(d) : a_n = \frac{\sqrt{n}}{\sqrt{n+1}}, \quad (e) : a_n = (-1)^n \frac{n}{n+1}, \quad (f) : \frac{n^2 + 1}{n^2 + n \ln(n)}.$$


---

**Problem 11 [8pts]:** Determine the convergence or divergence of the following series. Indicate which test you are using:

$$(a) : \sum_{n=1}^{\infty} \frac{2n}{n+7}, \quad (b) : \sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n, \quad (c) : \sum_{n=1}^{\infty} 2n^{-2/3}, \quad (d) : \sum_{n=1}^{\infty} \frac{n2^n}{n!}.$$