Formulas/Constants: PHYS/OCEA 4412/5412 A Atmospheric Dynamics II

$\Gamma_d = 9.8$ °C/km (dry adiabatic lapse rate)

$\gamma = c_p/c_v = 1.4$

R = 287.05 J/(kg K)

c_v = 717.5 J/(kg K)

c_p = 1004.5 J/(kg K)

g_0 = 9.81 m/s^2

$f = 2\Omega \sin\phi_0$

$\Omega = 7.292 \times 10^{-5}$ rad/sec

$p_0 = 1013.25$ hPa

$M_d = 28.94$ g/mole (Mean molecular weight of dry air)

$\nu = 1.46 \times 10^{-5} m^2/sec$ (sea level)

$\Gamma_d = 9.8$ °C/km

$a = 6370$ km (radius of the earth)

$\theta = T(p_s/p)^{\frac{(\gamma-1)}{\gamma}} = T(p_s/p)^{0.286} = T(p_s/p)^{R/c_p}$, where $p_s = 10^5$ Pa

Ideal Gas Law : $p = \rho RT$

specific volume: $\alpha = 1/\rho$

Pressure Gradient Acceleration : $\mathbf{F}/m = -\frac{1}{\rho}\nabla p = -\bigtriangledown_p \Phi$

Definition of Geopotential : $d\phi = gdz$

Hydrostatic Relationship : $dp/dz = -\rho g$

Thickness Relationship : $dz = -\frac{R<T>}{g} d\ln p$

Total vs Local Derivative : $d/dt = \partial/\partial t + \mathbf{U}\cdot\nabla$

Momentum Equation : $d\mathbf{U}/dt = -2\mathbf{\Omega}\times\mathbf{U} - (1/\rho)\nabla p + \mathbf{g} + \mathbf{F_r}$

Geostrophic wind : $\mathbf{V_g} = \mathbf{k}\times(1/(\rho f))\nabla p$

$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y}$$

$$v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

Momentum Equation p surface : $d\mathbf{V}/dt + f\mathbf{k}\times\mathbf{V} = -\nabla_{\mathbf{p}}\phi$

pressure velocity : $\omega = dp/dt$

Geostrophic wind on pressure surface :$\mathbf{V_g} = \mathbf{k}\times(1/f)\nabla_{\mathbf{p}}\phi$

In component form:

$$u_g = -\frac{1}{f}\frac{\partial\phi}{\partial y}$$

$$v_g = \frac{1}{f}\frac{\partial \phi}{\partial x}$$

$$\text{Continuity equation}: \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0$$

$$\text{Continuity equation}: \frac{1}{\rho}\frac{d\rho}{dt} + \nabla \cdot U = 0$$

$$\text{Thermodynamic equation}: c_v\frac{dT}{dt} + p\frac{d\alpha}{dt} = J$$

$$\text{Thermodynamic equation}: c_p\frac{dT}{dt} - \alpha\frac{dp}{dt} = \dot{Q}$$

$$\frac{T}{\theta}\frac{\partial \theta}{\partial z} = \frac{\partial T}{\partial z} + \frac{g}{c_p} = -\Gamma + \Gamma_d$$

$$v_g = \frac{1}{f}\frac{\partial \phi}{\partial x}$$

$$u_g = -\frac{1}{f}\frac{\partial \phi}{\partial y}$$

$$\text{Continuity equation in pressure coordinates}: \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$$

$$\text{Thermodynamic equation in pressure coordinates}: \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \omega S_p = \frac{J}{c_p}$$

$$\text{Static stability}: S_p = -\frac{T}{\theta}\frac{\partial \theta}{\partial p} = -\frac{\partial T}{\partial p} + \frac{RT}{c_p p} = \frac{\Gamma_d - \Gamma}{\rho g}$$

$$\text{Thermal Wind}: p\frac{\partial v_g}{\partial p} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p$$

$$p\frac{\partial u_g}{\partial p} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p$$

Other forms involving Φ

$$f_0\frac{\partial v_g}{\partial p} = \frac{\partial^2 \phi}{\partial x \partial p}$$

$$f_0 \frac{\partial u_g}{\partial p} = -\frac{\partial^2 \phi}{\partial y \partial p}$$

$$\text{Absolute vector vorticity} : \omega_{\mathbf{a}} = \bigtriangledown \times \mathbf{U_a}$$

$$\text{Relative vector vorticity} : \omega = \bigtriangledown \times \mathbf{U}$$

$$\text{Relative scalar vorticity} : \xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\text{Total (Absolute) vorticity} : \eta = \xi + f$$

$$\text{Thermal Wind in Vector Form} : \mathbf{V}_T = \frac{R}{f}\mathbf{k} \times \nabla < T > \ln(p_0/p_1)$$

$$u_T = -\frac{R}{f}\left(\frac{\partial < T >}{\partial y}\right)_p \ln(\frac{p_0}{p_1})$$

$$v_T = \frac{R}{f}\left(\frac{\partial < T >}{\partial x}\right)_p \ln(\frac{p_0}{p_1})$$

$$u_T = -\frac{1}{f}\frac{\partial(\phi_1 - \phi_0)}{\partial y}$$

$$v_T = \frac{1}{f}\frac{\partial(\phi_1 - \phi_0)}{\partial x}$$

$$\omega \approx -\rho g w$$

$$\text{Kinematic Estimate of } \omega : \omega(p) = \omega(p_s) - \int_{p_s}^{p} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp$$

$$\text{Surface Pressure Tendency} : \frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} (\nabla \cdot \mathbf{V}) dp$$

$$\text{Relative scalar vorticity} : \xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\text{Total (Absolute) vorticity} : \eta = \xi + f$$

$$\text{Ertel Potential Vorticity} : P = (\xi_\theta + f)\left(-g\frac{\partial\theta}{\partial p}\right)$$

$$\xi_g = \frac{1}{f_0}\nabla^2\Phi$$

$$\text{QG Vorticity Equation(5.45)} : \frac{\partial \xi_g}{\partial t} = -\mathbf{V_g}\cdot\nabla(\xi_g + f) + f_0\frac{\partial\omega}{\partial p}$$

Adiabatic QG Energy Equation (5.48) :

$$\frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right) = -\mathbf{V_g}\cdot\nabla\left(\frac{\partial\Phi}{\partial p}\right) + \sigma\omega$$

$$\sigma = \frac{RS_p}{p} = -\alpha\frac{\partial\, ln\theta}{\partial p}$$

Vorticity Equation Pressure Surfaces :

$$\frac{\partial\xi}{\partial t} = -\mathbf{V}\cdot\nabla(\xi + f) - \omega\frac{\partial\xi}{\partial p} - (\xi + f)\nabla\cdot\mathbf{V} + \mathbf{k}\cdot\left(\frac{\partial\mathbf{v}}{\partial p}\times\nabla\omega\right)$$

$$\mathbf{V}_{ag} = \frac{\mathbf{k}}{f}\times\frac{d\mathbf{V}}{dt}$$

$$\frac{d\mathbf{V}}{dt} - \left(\frac{d\mathbf{V}}{dt}\right)_0 = (\mathbf{V}_s\cdot\nabla)\mathbf{V}_0 + \frac{d\mathbf{V_s}}{dt}$$

$$\mathbf{V}_{ag,U} - \mathbf{V}_{ag,L} = \frac{\mathbf{k}}{f}\times\left[(\mathbf{V}_s\cdot\nabla)\mathbf{V}_0 + \frac{d\mathbf{V_s}}{dt}\right]$$

$$\mathbf{V}_{ag,I} = -\frac{1}{f^2}\nabla\frac{\partial\,\Phi}{\partial t}$$

$$\nabla\cdot\mathbf{V}_{isal} = -\frac{1}{f^2}\nabla^2\frac{\partial\,\Phi}{\partial t}$$

$$\mathbf{V}_{IA} = \frac{\mathbf{k}}{f}\times\mathbf{V_g}\cdot\nabla\mathbf{V_g}$$

$$f_0\left(\nabla\cdot\mathbf{V} - \nabla\cdot\mathbf{V}_0\right) = -\mathbf{V}'\cdot\nabla\left(\xi_{g0} + \xi_g + f\right)$$

$$\text{QG } \omega \text{ Equation} : \sigma\left(\nabla^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = f_0\frac{\partial}{\partial p}\left[\mathbf{V}_g \cdot \nabla(\xi_g + f)\right] + \nabla^2\left[\mathbf{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

$$\mathbf{Q} = (Q_1, Q_2) = \left(-\frac{R}{p}\frac{\partial \mathbf{V}_g}{\partial x} \cdot \nabla T, -\frac{R}{p}\frac{\partial \mathbf{V}_g}{\partial y} \cdot \nabla T\right)$$

$$\sigma\left(\nabla^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\,\nabla \cdot \mathbf{Q}$$

In a natural coordinate system where the isotherms are parallel to the s direction with the warm air on the right:

$$\mathbf{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial n}\right|\left(\mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial s}\right)$$

$$\mathbf{Q} = f\gamma\frac{d}{dt_g}\nabla\theta$$

$$\Im = \frac{d|\nabla_p\theta|}{dt}$$

$$\Im_x = \frac{d}{dt}\left(\frac{\partial \theta}{\partial x}\right)$$

$$\Im_x = \frac{d}{dx}\left(\frac{d\theta}{dt}\right) - \frac{\partial u}{\partial x}\frac{\partial \theta}{\partial x} - \frac{\partial v}{\partial x}\frac{\partial \theta}{\partial y} - \frac{\partial \omega}{\partial x}\frac{\partial \theta}{\partial p}$$

$$\Im_{2D} = \frac{-1}{2|\nabla\theta|}\left[D\left((\frac{\partial \theta}{\partial x})^2 + (\frac{\partial \theta}{\partial y})^2\right) + F_1\left((\frac{\partial \theta}{\partial x})^2 - (\frac{\partial \theta}{\partial y})^2\right) + 2F_2\left(\frac{\partial \theta}{\partial x}\frac{\partial \theta}{\partial y}\right)\right]$$

$$\Im_{2D} = \frac{|\nabla\theta|}{2}\left(F\cos 2\beta - D\right)$$

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$F_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

$$F_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$F = \left(F_1^2 + F_2^2\right)^{\frac{1}{2}}$$

β: angle between isentropes and local dilation axis

0°$< \beta <$45°: F promotes frontogenesis

45°$< \beta <$90°: F promotes frontolysis

$$\Im_{2D} = \left(\frac{1}{f\gamma}\right) \frac{\mathbf{Q} \cdot \nabla\theta}{|\nabla\theta|}$$

$$M_g = u_g - fy$$

Inertial Instability (with f constant): $\partial M_g / \partial y > 0$

$$\text{QG Geopotential Tendency Equation } \left(\chi = \frac{\partial \Phi}{\partial t}\right):$$

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi = -f_0 \mathbf{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) - \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\mathbf{V}_g \cdot \nabla\left(\frac{\partial \Phi}{\partial p}\right)\right]$$

$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p}$$

Penetration depth of PV anomaly: $H = fL/N$

$$\frac{dPV}{dt} \approx -g(\xi_\theta + f)\frac{\partial \dot{\theta}}{\partial p}$$

$$c = \overline{u} - \beta/k^2$$

$$\beta = \frac{\partial f}{\partial y}$$