PHYS/OCEA 4505/5505 A Atmospheric Physics Old Questions

Note: some of these questions refer to material no longer covered in the course. If something sounds totally unfamiliar, it probably will not be on a test.

=============== Chapter 6: General =====================

1. Use Rauolt's Law to determine the relative humidity e/e_s above the ocean surface (assumed flat), where e_s is the Clausius Clapeyron saturated vapor pressure of pure water. Assume the atmosphere above the ocean is in equilibrium with ocean (i.e. condensational flux equals evaporative flux at the ocean surface), that the only species dissolved in the ocean is NaCl, and that the ocean contains 3.3 kg of NaCl for every 100 kg of water. Na has a molecular weight of 23 g/mole, Cl has a molecular weight of 35 g/mole, and water has a molecular weight of 18 g/mole.

2. (i) What is the adiabatic liquid water content of an air parcel in a cloud?

The adiabatic liquid water content is the water content an air parcel would have if lifted adiabatically from cloud base. Technically speaking, it should be called a "density" since it is the local mass of liquid water per unit volume in a cloud, usually given in grams/m^3*. Adiabatic means that the air parcel has not experienced mixing or precipitation during ascent from cloud base. The text sometimes denotes LWC by* w_l*. However, it is better to use* w_l *to refer to the condensate mass mixing ratio, and write* $LWC = w_l \rho_d$*, where* ρ_d *is the local dry air density. Observed LWC values will lie between the pseudoadiabatic (all condensate removed) and adiabatic total water extremes. The recipe for determining the local adiabatic LWC is as follows:*

(i) given the cloud base temperature, calculate e_s *at cloud base,*

(ii) Use the cloud base pressure to find w_s *at cloud base,*

(iii) This gives w_t *at cloud base (*$w_t = w + w_l$*), since* w_l *is zero at cloud base and* $w = w_s$*,*

(iv) Adiabatic means w_t *is constant within the cloud,*

(v) At any point in the cloud above cloud base, use the local temperature to find w_s*, and get* w_l *from* w_t*,*

(vi) Multiply by the local dry air density ρ_d *to find the adiabatic LWC.*

(ii) Within a cloud, make a plot of how you would expect the adiabatic liquid water content to vary with height above cloud base. Also show on this plot a curve representing the actual mean liquid water content in the cloud.

(iii) What are two reasons why the adiabatic liquid water content and the actual mean liquid water content might be different in a cloud.

The two main reasons are mixing with subsaturated air outside the cloud and precipitation.

3. An aircraft is flying through a cloud at a temperature of $T = 260$ K and pressure $p = 600$ hPa. The air in the cloud is saturated (i.e. $e = e_s(T)$). Adopt a simplified model of a cloud in which all air parcels enter the cloud at cloud base where the temperature is $T_b = 280$ K, the pressure is $p_b = 800$ hPa, and that all air parcels in the cloud rise without experiencing precipitation or mixing.

(i) Draw a schematic vertical profile of the variation of water vapor mass mixing ratio w, liquid water mass mixing ratio w_l, and total water w_t as a function of altitude z in the cloud. Let z be the vertical coordinate, and w, w_l, and w_t lie along the horizontal axis. Indicate cloud base height z_b on your plot.

(ii) Estimate the total water mass mixing ratio w_t at cloud base.

(ii) Estimate the liquid water mass mixing ratio w_l measured by the aircraft.

$w_l = w_t - w$ by definition, and to find w assume the air in the cloud is saturated.

4. Does the latent heat of vaporization increase or decrease with temperature? Explain. A diagram may be helpful.

The latent heat is defined as the difference in specific enthalpy between water vapor and water. Use the fact that the specific heat of water is larger than that of water vapor.

=== Chapter 6: CCN, Kohler curve, Cloud Droplet Activation (Non-Math) ====

1. In the formula sheet, there is a simplified expression for the $e_s(f_{H_2O}, r, T)/e_s(T)$ ratio of a CCN involving the parameters a and b. Derive this expression, starting from either of the two previous expressions for the $e_s(f_{H_2O}, r, T)/e_s(T)$ ratio. You may assume that the CCN are in the dilute, large r limit.

2. (i) Draw a typical Kohler curve of a CCN with a small dry mass, together with the Kohler curve of a CCN with a larger dry mass. Pay attention to the relative positioning of the critical radii r_{crit} and the critical supersaturation ratio S_{crit} of the two curves. The y axis should refer to the supersaturation $S = (e - es)/es$, while the x axis should refer to the radius of the CCN.

(ii) In general, would you expect the small or large CCN to be more likely to experience activation into a cloud droplet near cloud base. Explain.

3. The likelihood that a CCN will activate into a cloud droplet will, in general, depend on the number and size of the other CCN in the air parcel. Explain. (You may find a diagram helpful.)

Was looking for a Kohler curve for a large and small CCN, with explanation that CCN with smaller S_c which activate first will subsequently inhibit other CCN from activating.

4. CCN that do not activate when an air parcel originally becomes cloud usually do not activate at any subsequent time. There is, however, an exception.

(i) In what type of circumstance does this exception occur? (NOT DONE)

(ii) Explain how CCN can activate even though cloud droplets are present. In your explanation, show a plot of how the percent supersaturation s in a cloudy air parcel varies with time, starting before the original activation of the CCN, during the original activation, and then following the behaviour of s with time during the second stage of CCN activation. Explain the behaviour of s at each stage. (NOT DONE)

5. Not all CCN are activated during cloud formation. What is one property of a CCN which has a big impact on whether a CCN is activated, and explain why it has this effect. To answer this question, please draw a Kohler curve for a small CCN and a large CCN, and label the critical radius r^* and the critical Supersaturation ratio S^* in each case.

Larger CCN have a larger r_{crit} and smaller S_{crit}. The smaller S_{crit} of the larger CCN means they will activate before the smaller CCN. Once activated into cloud droplets, the cloud droplets will keep the ambient RH near 100 percent, and inhibit subsequent activation of CCN.

6. What is the likelihood that an unactivated CCN will become activated in the presence of other cloud droplets? Explain why or why not.

Unactivated CCN have essentially no probability of being activated once cloud droplets are present, since cloud droplets will keep the RH in a narrow range about 100 percent. CCN can become incorporated into cloud droplets by colliding with a cloud droplet. However activation refers to a sudden increase in size of a CCN due to rapid condensational growth.

7. Would you expect more CCN to be activated in a slow or a fast cloud updraft? Explain.

The logic goes as follows: faster updraft speed $\rightarrow$ faster expansion cooling $\rightarrow$ faster reduction in $e_s(T)$ $\rightarrow$ faster increase in S above cloud base $\rightarrow$ higher max S achieved near cloud base $\rightarrow$ more of the smaller CCN activated. In reality, during the cloud activation process, there is a complex competition between the removal of water vapor due to rapid condensational growth (which is lowering S), and the tendency for S to increase due to the updraft velocity, but this naive argument should have some validity.

8. In what size range does the droplet growth rate become unstable with respect to perturbations in radius? Explain the origin of this instability. Explain with reference to a Kohler curve.

Referring to the size range where $r > r_c$.

==== Chapter 6: Cloud Droplet Condensational Growth Non-Math =====

1. (i) Plot the variation in temperature with distance R from a cloud droplet experiencing condensational growth. Assume the temperature at the surface of the drop is T_r, with r the radius of the drop.

(ii) Suppose the droplet is evaporating. Would the temperature decrease or increase as you move away from the drop? Explain.

If the droplet is evaporating, then it will cool the surface of the cloud drop (evaporating sweat cools your skin). The temperature will increase as you move away from the drop (as it must in order for evaporative cooling of the drop to be balanced by diffusion of heat toward the drop.)

2. Suppose a droplet is shrinking in a slightly undersaturated environment (i.e. S is a bit less than 0). Let R represent the distance from the center of the droplet. The radius of the drop is r.

(i) Plot how the temperature T would vary as a function of distance R from the droplet. Let T_r represent the temperature at the surface of the droplet.

(ii) Plot how the water vapor density n would vary as a function of distance from the droplet. Let n_r represent the water vapor density at the surface of the droplet.

3. (i) A cloud droplet is experiencing condensational growth in a cloud. Would you expect the temperature of the cloud droplet to be higher, lower, or the same, as the average temperature of the air in the cloud. Explain. (Ignore radiative effects.)

(ii) In class, we derived an expression for the rate dr/dt of condensational growth of a cloud droplet in which it was assumed that the temperature of the cloud droplets was the same as the temperature of the cloud. However, if the temperature effect were taken into account, how would this affect the rate of condensational growth dr/dt: would you get a larger dr/dt, smaller dr/dt, or the same? Explain.

4. (i) Plot how the supersaturation $S = (e - e_s)/e_s$ typically depends on height in an air parcel as it rises above cloud base. Show this dependence for two cases: (a) one with an updraft speed of 2 m/sec, and (b) with an updraft speed of 0.5 m/sec.

Below the cloud RH < 1 *so* $S < 0$*, but both are increasing with height (due to expansion cooling and reduction in* e_s*). At some point an air parcel achieves RH* $= 1$*, or* $S = 0$*. However, no activation of cloud droplets occurs at this point because even large CCN require some* $S > 0$ *to achieve activation. However, at some some positive value of* S*, the larger CCN start to activate. By absorbing water vapor, will attempt to drag* S *to zero. A vertical profile of* S *would therefore have* $S < 0$ *initially, increasing with height to some small positive value (maybe about 0.01), and then decreasing toward zero as you go above cloud base.*

(ii) For each of the two cases given above, plot how the cloud droplet concentration depends on height.

(iii) Why is the number of cloud droplets formed in an updraft sensitive to the updraft speed?

see previous discussion

6. Condensational growth of cloud droplets tends to produce a mono-disperse size spectrum, i.e., one in which all cloud droplets have a similar size. Briefly discuss one physical process that tends to broaden the cloud droplet size spectrum.

7. What are the two main ways in which the growth of droplets in a cloud can affect the updraft speed of the cloud?

The effects on updraft speed would be mediated through the effects of droplets on cloud buoyancy. The two main direct effects would be:
(i) Latent heat release from condensation increases temperature and buoyancy.
(ii) Condensate loading (i.e. liquid water or ice mass mixing ratio) decreases buoyancy.

8. Does homogeneous nucleation of water droplets occur in the atmosphere? Why or why not?

Homogeneous nucleation of water droplets from the vapor phase is very unlikely in the atmosphere. You would have to generate extremely high relative humidity's (likely higher than 400 percent RH). The existence of CCN in the atmosphere provides surfaces for water vapor to condense, as soon as the RH exceeds 100 percent by fractions of a percent. You could also say that homogeneous nucleation is prevented by the curvature effect. Because e_s *goes up with smaller size, the nucleation of new small water droplets (clusters) directly from the gas phase is VERY hard.*

Warning: The term "homogeneous" nucleation is used in different ways. For example, homogeneous nucleation of ice usually refers to the freezing of liquid water droplets (usually containing some dissolved sulfuric acid) without an ICN present (occurs around -40 $^\circ$*C).*

9. (i) Can condensational growth make rain in a realistic time? Why or why not? (Include in your answer a plot of how the radius of a cloud droplet typically increases with time by condensational growth under a constant S.)

No, by itself, condensational growth cannot produce raindrop sized cloud particles in a realistic time (i.e. on a time comparable to the lifetime of a cloud, or on a timescale that clouds are observed to produce rain). This is because the rate of increase of the radius of a drop dr/dt goes as 1/r. r(t) therefore *scales with the root of time, so that dr/dt slows down as the radius of the cloud droplet gets bigger. Condensational growth dominates the initial growth of small droplets (r less than 10 microns).*

10. Specify and briefly discuss two of the approximations used in deriving the expression for condensational growth dr/dt in the notes.

Pick from: (i) isotropic water vapor distribution around growing cloud droplet
(ii) cloud droplet at same temperature as the atmosphere
(iii) water vapor field around the droplet in steady state
(iv) cloud droplets spherical

11. In general, would you expect cloud droplets to be larger near the top of a cloud or near the bottom of a cloud? Explain.

As an air parcel in a cloud rises, the temperature and e_s go down. The water vapor mass mixing ratio w therefore decreases with height. Provided the total water mass mixing ratio w_t is conserved (or nearly conserved), the liquid water vapor mass mixing ratio should increase with height in the cloud. As a result, cloud droplets should grow by condensation as long as an updraft speed is maintained (and loss of w_t via precipitation is weak).
In the case of a precipitating cloud, you would expect the larger raindrops/drizzle drops to occur near cloud base (i.e. growing by collection as they fall, so larger at the bottom).

=== Chapter 6: Cloud Droplet Condensational Growth: Math Questions ===

1. (i) A spherical water droplet is growing by vapor deposition in a cloud that has a supersaturation S. Using the solutions for condensational growth in the formula sheet, show that the rate of change of mass of the water droplet can be expressed in terms of a diffusion constant D, the radius r of the droplet, the background water vapor density of the cloud $\rho_v(\infty)$.

$$\frac{dm}{dt} = 4\pi r D \rho_v(\infty) S$$

(ii) Assume that all of the heat released during vapor deposition is used to increase the temperature of the water droplet. Show that the rate of change of temperature dT/dt of the droplet is given by the following relationship, where ρ_w is the density of liquid water. (Hint: what is the relationship between the heat dQ added to a droplet and the temperature change dT?).

$$\frac{dT}{dt} = \frac{3 l_v D \rho_v(\infty) S}{c_l \rho_w r^2}$$

(iii) Use this expression to calculate the heating rate dT/dt in K/s of a droplet experiencing condensational growth. Assume that S = 0.005, r = 20 μm, p = 600 hPa, T = 275 K, and the diffusion coefficient of water vapor in air can be expressed $D = 21.2 \times 10^{-6}(1+0.0071\, T_c)$ with the temperature T_c in Celsius, and D having units of m^2/s.

Note that this solution would be somewhat inconsistent, in the sense that it would include the molecular diffusion of water vapor toward the growing droplet, but ignore the thermal diffusion of heat away from the droplet.

Chapter 6: Ice Condensation Nuclei (ICN)

1. Specify and briefly discuss two modes of operation of an ice condensation nucleus (ICN).

Freezing mode and deposition mode.

2. What are two physical variables of a cloud that affect its ICN concentration? How do these two variables affect the ICN concentration? (Note: not looking for factors that might affect the chemical composition of the air in the cloud).

include: age of cloud, cloud top temperature, S_i, ice splintering.

3. In general, the ability of an Ice Condensation Nucleus (ICN) to nucleate an ice crystal depends on both the ice supersaturation $S_i = (e - e_{si})/e_{si}$ and temperature. In the S_i versus temperature plane, roughly show the regions in which a typical ICN such as silver iodide (AgI) will:
(i) function as a heterogeneous freezing nucleus,
(ii) function as a vapor deposition nucleus
(iii) fail to nucleate ice.
Allow S_i to vary from 0 to 0.3 on the y axis, and temperature to vary from 0 to -25 °C on the x axis. Indicate the water saturation curve ($e = e_s$) on the diagram.

This question concerns Figure 6.30. However this figure is confusing for a variety of reasons, and I would not likely ask a student to reproduce it in the future. First, the maximum value of S_i occurs when you have $e = e_s$ (since water deposition nuclei in the atmosphere are common). Therefore, you can't go above the diagonal line denoted "water saturation". I don't know why they have labeled this region "Condensation-freezing". It is also confusing to have the reversed temperature scale. Finally, S_i with water saturation increases with colder temperature non-linearly, not linearly as they suggest. I think it is better to show a region where a specific deposition ICN works by indicating this region in the $e - T$ plane, as I have in the notes. Overall, it is important to understand that the likelihood of an ICN starting to initiate ice formation will be a function of S_i, T, and also time (since a non-equilibrium process). If we ignore time, then we could cross into the "activation region" for an ICN either by increasing S_i or decreasing T. Therefore between the e_s and e_{si} curves, a dashed line denoting the activation region of a deposition ICN would start somewhere on the e_s curve and then slope downward toward colder temperatures, as in the notes.

4. Show a schematic plot of the region in the e versus T plane where an ice deposition nucleus would typically be able to nucleate the formation of ice from the vapor phase. In this plot, indicate the saturation vapor pressure curves for water and ice (i. e. $e_s(T)$ and $e_{si}(T)$).

The possible region for a deposition ICN to nucleate a new ice crystal is between the e_{si} and e_s curves. I gave partial marks to people who drew this entire region. However, no real ICN will operate in this entire region. As discussed in class, a deposition ICN will more likely initiate ice formation as T goes down and S_i goes up. Therefore, a typical region will start somewhere along the e_s curve and slope down and to the left toward colder temperatures. Some drew the diagram in the text Fig 6.30. However, this is not what the question asked. I am de-emphasizing this diagram because students find it so confusing. It is better to draw this region in the $e - T$ plane.

5. A mixed phase cloud contains both water droplets and ice crystals. How would you typically expect the sizes of the water droplets and ice crystals to be changing with time? Explain.

======== Chapter 6: Saturation Vapor Pressure over Ice =======

1. Show a schematic plot of the variation of the supersaturation ratio with respect to ice S_i in a mixed phase cloud between -40 °C to 0 °C. S_i should be on the vertical axis. Give a rough number for S_i at -40 °C.

2. Show a schematic plot of the variation of $e_s - e_{si}$ between -40 °C to 0 °C. Indicate the approximate temperature at which this vapor pressure difference has its maximum value.

3. (i) Plot how the difference between the saturation vapor pressures for water and ice (e_s - e_i) varies with temperature between -40 °C and 0 °C.

(ii) Explain how this affects the temperature dependence of the growth rate of ice crystals in a mixed phase cloud.

===== Chapter 6: Freezing of Supercooled Water ==============

1. A supercooled water droplet at -20 °C starts to freeze. What fraction of the droplet will be frozen if the final temperature of the droplet (including the ice) is 0 °C, and there is no transfer of heat between the droplet and the surrounding atmosphere.

2. (i) Homogeneous freezing of supercooled cloud droplets is usually a two stage process. Why?

The heat release from freezing makes the remainder of the water warm up, possibly to 0 °C, so prevents freezing of the entire droplet. Very small droplets can get quickly get rid of this "excess" heat by diffusion, so the two stages might not be separated by very much in time.

(ii) Estimate the threshold temperature at which a supercooled droplet will first typically freeze rapidly to completion.

Use conservation of enthalpy.

3. What observations suggest that the homogeneous freezing of pure supercooled water droplets is rare?

Homogeneous freezing of pure supercooled water droplets implies freezing without the help of a freezing ICN. This does not ordinarily occur until temperatures close to -40 °C, where the free energy barrier between supercooled water and ice is small. But most clouds will freeze at temperatures warmer than -40 °C (very strong updrafts being an exception).

========= Chapter 6: Aircraft Icing and Contrails ==============

1. Aircraft icing can sometimes be reduced by flying to a higher altitude. Explain.

Aircraft icing is caused by supercooled water hitting the plane. The supercooled water can come either from supercooled cloud droplets or supercooled rain. Supercooled cloud droplets are generally rare below -20 °C, since ICN are more common below -20 °C. So, if a plane is flying through a mixed phase cloud, then the amount of supercooled water in the cloud should become smaller if the plane flies higher to a colder temperature where there is more ice. Ice will just bounce off the plane, and not interfere with its aerodynamics.

There have been reports of supercooled water in convective updrafts at temperatures close to -40 °C. Probably in this case, within a very strong updraft, it is easier for supercooled water to reach these high altitudes because there is less time for the ICN to act, or perhaps if there are fewer ice crystals, less time for the Bergeron process to reduce the amount of supercooled water.

2. In what season (winter or summer) would you expect contrails from jet aircraft flying near 11 km to be long-lived most frequently? Explain.

Jet exhaust contains water and ICN. Any oxidation of hydrocarbons will produce water vapor. I am not sure what the ICN are. Some part is likely coming from the sulfur in the fuel; likely also some black carbon. If the exhaust is injected into a region where $S_i < 0$, the exhaust could make short-lived contrails simply because they are a source of water, causing $S_i > 0$ locally. Note that e_{si} is extremely small at these cold temperatures, so the amount of water required to produce $S_i > 0$ locally is not very large. However, the contrails produced from this mechanism will be relatively short-lived and narrow, being destroyed by mixing with the background atmosphere fairly quickly. However, if $S_i > 0$ in the background atmosphere (not that uncommon in the upper troposphere due to the scarcity of ice deposition nuclei and because synoptic scale ascent is not uncommon), then the exhaust could produce a cirrus cloud over a larger region by supplying ICN. In this case, horizontal mixing will cause the contrail to spread horizontally, and falling ice crystals can nucleate more cirrus below the contrail, provided $S_i > 0$ there also.

============ Chapter 6: Probability of Ice in a Cloud ==========

1. Plot how the likelihood that a cloud will contain ice depends on the cloud top temperature. Let the cloud top temperature vary between 0 C and -40 °C. Indicate on this plot the temperatures at which clouds tend to be all ice or all water.

============ Chapter 6: Fall Velocity ===============

1. A small ice crystal starts at the top of a mixed phase stratiform cloud at a temperature of -20 °C. It eventually reaches the surface where the temperature is 20 °C. The base of the cloud is at 0 °C. Show how the terminal velocity of the condensed particle (ice or rain) would typically vary with height in going from the initial to the final temperature. Do not do any math. Just show a plot. Assume that the ice crystal experiences growth or loss processes typical of these conditions. The vertical axis should of your plot should refer to temperature with -20 °C at the top, and 20 °C at the bottom. The horizontal axis of your plot should refer to the terminal velocity. Give rough numbers for the terminal velocity. Explain as much of the vertical variation in terminal velocity as you can.

As an ice crystal falls toward the melting level, it will typically grow by condensation and aggregation, and reach a falls speed of 1-2 m/s. When it melts into a raindrop, it will jump to about 8 m/s. As raindrops fall, the fall speed will get smaller due to the increase in atmospheric density (air friction), and possible evaporation (which makes the raindrop smaller). A typical value near the ground might be 5-6 m/s.

==== Chapter 6: Condensational Growth of Ice Crystals, Non-Math ====

1. In class, we derived an expression for the rate of condensational growth of an ice crystal, using a method that was similar to that used to find the condensational growth of a water droplet. However, this expression tends to overestimate the real growth rate of ice crystals by about a factor of two. Why?

2. Ice crystals can become precipitation sized via condensational growth, whereas cloud droplets typically do not. Explain.

In both water droplets and ice crystals, the rate of condensational growth is proportional to the supersaturation S. In water clouds S rarely exceeds 0.01 (1 percent supersaturation.) In ice clouds, especially mixed phase clouds, values of Si can be as high as 0.4.

3. Condensational growth of ice crystals in mixed phase clouds is much faster than condensational growth of cloud droplets in pure water clouds. Why?

4. In a mixed phase cloud, ice crystals can grow by either vapor deposition or accretion. What is the main property of a cloud that favors growth by accretion over depositional growth? Under what conditions is this property more likely to be realized?

5. In a mixed phase cloud containing both water droplets and ice crystals, how would you typically expect the sizes of the water droplets and ice crystals to be changing with time? Explain.

6. Suppose the thermal conductivity of the atmosphere were to increase. Would you expect the condensational growth rate of ice crystals to increase or decrease? Explain.

In order for ice crystals to grow by vapor deposition, the condensation of water vapor molecules next to the surface of a growing ice crystal must continuously be resupplied by diffusion of water molecules toward the ice crystal from the background atmosphere. In addition, the heat released as water vapor condense on the ice surface must be continuously conducted away from the crystal to prevent the ice from heating up and melting, or having its temperature increase to the extent that the evaporation of ice molecules from its surface increases and condensational growth stops. Ice condensational growth is limited by both of these processes. If you increase the thermal conductivity of the atmosphere, you make it easier for a growing ice crystal to dissipate heat, and it will grow faster.

7. Indicative in a qualitative way (no numbers) how the growth rate dm/dt of an ice crystal growing in a water saturated environment typically varies as a function of temperature between 0 C and -40 C. In your plot, point out the limits at 0 C and -40 C, and discuss the reasons for the behaviour in the growth rate in these two limits.

8. Specify and very briefly define the three main growth processes of ice crystals in clouds.

========= Chapter 6: Growth of Ice Crystals, Math ===========

1. This question involves the condensational growth of an ice crystal. The crystal is a flat circular disk of radius r and constant thickness h, with capacitance $C/\epsilon_0 = 8r$.

(i) Derive an expression for the growth rate dr/dt of the crystal in terms of G_i, S_i, π, h, the density of ice ρ_i (and any other possible variables).

(ii) Assume the cloud is mixed phase at -10 °C. What is S_i? The vapor pressure of ice (in mb) is given by $e_{si} = e_{so} exp[22.49 - (6142/T)]$, where T is in K, and $e_{so} = 6.11$ mb.

(iii) Let $G_i = 1 \times 10^{-9}$ kg/(s m), crystal thickness $h = 10\,\mu m$, and density of ice $\rho_i = 917\,kg/m^3$. Suppose the initial radius of the disk is 10 μm. What will be the radius after 30 minutes of growth?

2. For a circular disk of ice experiencing condensational growth, $C/\epsilon_0 = 8r$. Assume that the thickness h of the disk is fixed at $h = 10\mu m$, so that condensational growth gives rise to a non-zero dr/dt. REPLACE BY VERSION IN ex.tex

(i) Express the mass m of the disk in terms of the radius r, thickness h, and ice density ρ_i.

(ii) Show that the rate of change of radius of the disk can be written.

$$\frac{dr}{dt} = \frac{4D\rho_v(\infty)S_i}{\pi h \rho_i}$$

(iii) The disk is in a mixed phase cloud (i.e. saturated with respect to water) at a temperature of -5 °C. What is S_i?

(iv) What is the background water vapor density in the cloud $\rho_v(\infty)$?

(v) The diffusion coefficient of water vapor in air can be expressed $D = 21.2 \times 10^{-6}(1 + 0.0071\, T_c)$ with the temperature T_c in Celsius, and D having units of m^2/s. Solve for dr/dt.
Should get $dr/dt = .44\mu m/s$.

(v) The disk is in a cloud with an updraft speed w. The terminal fall speed of the disk is given by $\nu(r) = kr$, where $k = 6000\, s^{-1}$. Write down an expression for the rate of change of height dh/dt of the disk in terms of w, k, and r, where h is the height above cloud base, which is fixed. (Hint: is not a hard question.)

(vi) Assume $w = 2$ m/s, and the initial radius of the disk is $r_i = 20\mu m$. The disk is initially 1 km above cloud base. Calculate the final radius r_f of the disk when it exits the cloud.
Answer: There are a variety of ways of doing this question. The method below is longer than neccessary, but goes through the up+down steps in detail.
Upward: *The initial terminal fall speed is is $\nu = 6000 \times 0.00002 = .12$ m/s. So, initially the disk will be lifted vertically by the updraft. It will continue to rise until $dh/dt = w - \nu = w - kr = 0$, at which point the radius will be $r_{top} = w/k = 333\,\mu m$. To solve for the altitude gained during this part of the trajectory, use $dh/dt = w - kr(t)$. You know that dr/dt is a constant, solved earlier to be $.44\,\mu m/s$. $r(t)$ therefore increases linearly with time: $r(t) = r_i + (dr/dt)t$. Using $r_i = 20\mu m$ and $r_{top} = 333\,\mu m$, gives a time to reach the top of 711 s.*
Now use $dh/dt = w - k\,r(t)$, with $r(t)$ given by the linear expression, to show that $dh = (w - kr_i)dt - k(dr/dt)tdt$. Integrate this from $t = 0$ to $t = 711$ s. You should get a height increase of 670 m.
Downward: *You now have a new problem, where a disk of initial radius of $r_{top} = 333\mu m$ is falling a total distance of (670 + 1000) m through a cloud at a constant growth rate of $.44\,\mu m/s$.*
You again use $dh = (w - kr_i)dt - k(dr/dt)tdt$, where the integrated height and r_i are known, to solve for the time. In this case, however, $w - kr_i = 0$, so $dh = -k(dr/dt)tdt$. This gives $h = -k(dr/dt)t^2 \times 0.5$. Keep in mind $h = -1670$ m. This gives a time of 1125 s. Starting with an initial radius at the top of 333 mum, and growing linearly at .44/mum/s, this gives an exit radius of 838 μm.

======== Chapter 6: Warm Rain processes, Non-Math =========

1. There are a variety of explanations for how collector sized droplets of radii greater than 20 μm come into existence. Name and briefly discuss two of these explanations.

extra large CCN to start and turbulence. Turbulence at cloud base would expose activating CCN to a range of S(t), (e.g. larger S at cloud base locations where the updraft speed was larger), and presumably increase the variance in cloud droplet sizes.

2. Most clouds do not generate precipitation. What are two factors that make it more likely for a given cloud to precipitate?

Higher updraft speed (i.e. stronger buoyancy), longer lifetime, higher background RH.

3. (i) Can condensational growth make rain in a realistic time? Why or why not? (Include in your answer a plot of how the radius of a cloud droplet typically increases with time by condensational growth under a constant S.)

4. A collector droplet of radius $r_1 = 20\mu m$ is falling through a cloud of cloud droplets of variable radii r_2. Make a rough plot of how you would expect the collision efficiency E of the collector drop to depend on r_2 for $r_2 \leq 20\ \mu m$.

5. A collector drop of radius r_1 is falling through a cloud of liquid water content w_l in which all the droplets have the same radius r_2. The terminal velocity of the droplets is ν_1 and ν_2 respectively. The collection efficiency is E_c, and the density of water is ρ_w. Write down an expression for the rate of change of the mass of the collector droplet, dM/dt. (Note: this does not involve a derivation)

6. Discuss two mechanisms by which turbulence is thought to initiate or accelerate collision/coalescence.

Turbulence can generate confluent air parcel trajectories, and bring cloud droplets closer together. It is also suspected that turbulence at the boundaries of a cloud may decrease the CD density, and increase average CD radius.

7. Plot how the collision efficiency E, between a collector drop of radius R and a 10 μm cloud droplet, would depend on R. Roughly plot E versus R for R between 0 and 100 μm.

Figure from the text.

8. The collision efficiency between raindrops of similar radii is approximately equal to one. Why?

9. Suppose two raindrops collide and stick together. What are the two types of energy that must be dissipated in order for the raindrop to stay together after the collision?

10. (i) In general, how is updraft velocity expected to affect raindrop size? What is the main reason for this dependence?

Higher updraft speed means larger radii RD at cloud base exit. This is addressed in the continuous collection model. Higher updraft velocity means growing CD remain partially suspended for longer periods in the cloud (i.e. and don't actually start falling till terminal fall speed equals updraft speed). Note: this is a different question from how you would expect the updraft velocity to affect cloud droplet sizes.

(ii) Suppose two cloud identical droplets are formed at the base of two clouds that are identical except that one has an updraft speed of 1 m/sec and the other has an updraft speed of 0.5 m/sec. Plot how the radii of the two cloud droplets would depend on time in the cloud from the moment of formation in the cloud, to when they exit out the cloud bottom. Draw the two curves on the same plot. The two droplets grow by collision/coalescence.

In notes as part of discussion of continuous collection model.

11. In the collision of two raindrops, discuss one factor (or variable) that helps determine the outcome of the collision, and especially whether or not the new combined droplet will be stable, or breakup.

You could discuss surface tension, the collision kinetic energy, the relative velocity, etc.

========== Chapter 6: Warm Rain Processes, Math ==========
========== Collector Drop Starting at Cloud Base ==========

1. A collector cloud droplet starts off at cloud base with a radius of 20 μm. The cloud has a constant upward vertical velocity of w = 1 m/s. The fall speed of the cloud droplet is given by $v_1(r) = k_3 r$, where where $k_3 = 6000$ /s. Assume the collection efficiency $E = 0.8$, and that the Liquid Water Content is $LWC = 2$ g/m^3. Assume the terminal velocities v_2 of the droplets being collected in the cloud can be set to zero. The droplet grows by collision/coalescence only in a way that can be described by the continuous collection model.

(i) What is the size of the cloud droplet in μm at its highest altitude in the cloud?

(ii) How long does it take for the droplet to reach the highest altitude from cloud base?

(iii) Derive and expression for dh/dr of the droplet in the cloud in terms of v_1, w_l, E_c, and ρ_w. (Hint: you need an expression for dh/dt).

(iv) Manipulate this expression for dh/dr to derive the expression for H (distance of the droplet above cloud base) in the formula sheets.

2. Within a cloud, there is an updraft speed w = 20 cm/s, a collision efficiency E = 0.8, and a cloud liquid water content LWC = 0.005 kg/m^3. A collector cloud droplet starts off at cloud base with r_0 = 40 μm. The cloud droplet has a velocity with respect to the air parcel given by $\nu_1 = kr_1$, where k = 1100 s^{-1}. You can assume that the velocity of the collector cloud droplet is much larger than the velocities of the other cloud droplets (i.e $\nu_1 >> \nu_2$).

(i) What is the radius of the collector droplet at its highest altitude in the cloud?

(ii) At the highest altitude, how far is the droplet above cloud base?

(iii) How large is the cloud droplet when it emerges from the cloud base?

========== Chapter 6: Warm Rain Processes, Math ==========
========== Collector Drop Starting at Cloud Top ==========

1. A drizzle droplet of radius r_0 = 0.05 mm starts at the top of a cloud that is 2 km thick. Assume that the air in the cloud has zero vertical velocity (i.e. w = 0). The vertical velocity of the drizzle droplet is given by $\nu(r) = k_3 r$, where $k_3 = 8 \times 10^3\, s^{-1}$, where r is the radius of the droplet. The cloud has a uniform liquid water content of $LWC = 1\, g/m^3$. Assume a collection efficiency E_c of unity. You can use the elementary form of the continuous growth equation. Neglect growth by condensation. The density of water is $\rho_w = 1000\ kg/m^3$.

(i) Let Δt be the time it takes to fall through the cloud, and r_f the drizzle droplet radius on exit. Show that r_f is related to Δt through,

$$\ln(r_f/r_0) = \frac{k_3 \cdot LWC \cdot \Delta t}{4\rho_w}$$

Straightforward.

(ii) Let h refer to the height of the drizzle drop in the cloud. Show that dh/dr can be expressed:

$$\frac{dh}{dr} = \frac{4\rho_w}{LWC}$$

Use $dh/dt = dh/dr \cdot dr/dt$, the expression for dr/dt from (i), and $dh/dt = k_3 r$ (since the updraft speed is zero) to solve for dh/dr.

(iii) What is the radius of the drizzle drop when it exits the cloud?

This is a straightforward integration of the expression for dh/dr in (ii).

(iv) How long does it take for the drizzle drop to fall through the cloud?

Plug the solution for the exit radius from (iii) into the expression in (i).

2. A drizzle droplet of radius $r_0 = 0.05$ mm starts at the top of a cloud with zero vertical velocity, and starts to fall through a cloud that is 2 km thick. Assume that the air in the cloud has zero vertical velocity (i.e. $w = 0$). The vertical velocity of the drizzle droplet is given by $\nu(r) = k_3 r$, where $k_3 = 8 \times 10^3\, s^{-1}$, where r is the radius of the droplet. The cloud has a uniform liquid water content of $LWC = 1g/m^3$. Assume a coalescence efficiency $\overline{E}$ of unity. You can use the elementary form of the continuous growth equation. Neglect growth by condensation. The density of water is $\rho_w = 1000\, kg/m^3$.

(i) Find the final size r_f of the droplet as it emerges from the bottom of the cloud.

(ii) Find the time Δt it takes for the droplet to fall through the cloud.

(iii) Now assume that the cloud has a uniform updraft speed of $w = 30cm/sec$. Find the final size r_f of the drop as it emerges from the bottom of the cloud. (Hint: you can get an exact answer for r_f, but it involves solving integrals, and then plugging in guesses for r_f, and seeing which works. Tough question; won't be asked on a test.)

========= Chapter 6: Raindrop Evaporation, Math ==========

1. (i) The radius r of a falling raindrop decreases due to evaporation at a rate given by $rdr/dt = G_l S$. Assume that a raindrop exits cloud base at altitude H with radius R. It falls with a radius dependent velocity given by $\nu(r) = kr$. Derive an expression for $r(h)$ in terms of R, G_l, S, k, H, and h, where h is some distance below cloud base. Assume that the vertical velocity w of the background atmosphere is zero.

This is very similar to an expression for $r(h)$ derived in class for the case $\nu(r) = k\sqrt{r}$, i.e. using expression for terminal velocity based on a parameterization of friction using a drag coefficient.

(ii) Assume that the relative humidity of the atmosphere is 60 %, $G_l = 7 \times 10^{-10}\, m^2/s$, $k = 6000\, s^{-1}$, $z_0 = 5000$ m. What is the critical radius R_{crit} for which raindrops will hit the ground for initial radius $R > R_{crit}$, and evaporate before hitting the ground for $R < R_{crit}$?

2. A raindrop exits a cloud at a distance H above the surface at $t = 0$. Its initial radius is R. Let h refer to the height of the raindrop above the surface (i.e. $h = H$ at $t = 0$). The fall velocity of the raindrop is given by $\nu(r) = kr$. Note that $\nu(r) = -dh/dt$. The atmosphere below the cloud is unsaturated (i.e. $S < 0$ or $RH = e/e_s < 1$). The raindrop therefore evaporates, with a decrease in radius given by

$rdr/dt = G_l S$. Assume that S is constant below the cloud. The vertical velocity of the background atmosphere is zero. Assume that the relative humidity of the atmosphere is 60 %, $G_l = 7 \times 10^{-10}\, m^2/s$, $k = 6000\, s^{-1}$, and $H = 2000$ m.

(i) Show that $r(h)$ can be expressed:

$$r(h)^3 = R^3 + \frac{3G_l S}{k}(H - h)$$

(ii) What is the critical radius R_{crit} such that droplets with $R > R_{crit}$ will reach the surface? Express your answer in mm.

3. A raindrop with a radius $r = 500\ \mu m$ exits a cloud and starts falling through unsaturated air below the cloud having a relative humidity of 0.6 (or 60 percent). Assume that the raindrop remains spherical as it falls, and that the rate of change of radius of the raindrop is given by $dr/dt = G_l S/r$. Assume a fixed temperature of $T = 280$ K, and the expression for D given in the previous question.

(i) If the raindrop were to completely evaporate before it hit the ground, how long would this take? Assume a fixed temperature of $T = 280$ K, and the expression for D given in the previous question.

(ii) Assume that the raindrop falls with a terminal velocity $\nu(r) = kr$, where $k = 6000\, s^{-1}$. How far would it fall before it is evaporated?

Answer: know that the distance is $h_{evap} = \int_0^{t_{evap}} (dh/dt)dt$. So

$$h_{evap} = \int_R^{r=0} (dh/dt)(dt/dr)dr = -(k/G_l S) \int_R^{r=0} r^2 dr$$

The integral is now solvable for the evaporation distance h_{evap} as a function of the initial raindrop size R.

============ Chapter 6: Precipitation Forecasting ==============

1. Weather forecast models have difficulty in predicting precipitation.

(i) Give one reason why an error in a precipitation forecast may give rise to a subsequent error in the wind or pressure fields of a forecast model.

(i) Give one reason why an error in the wind or pressure fields of a model may give rise to an error in a precipitation forecast.

2. Much of this class is devoted to trying to understand how to predict under what circumstances a cloud might precipitate. There are a number of reasons why this is a difficult problem.

(ii) What is one reason why it is difficult to create simple models of this process?

(ii) What is one reason why it is difficult to test from observations the predictions or validity of theoretical treatments of cloud physics in real clouds?

One reason is that water vapor mixing ratios and temperatures cannot be measured with sufficient accuracy and time resolution to calculate the small supersaturations S that occur in liquid water clouds. This is unfortunate, since S is the variable which drives condensational growth.

============ Chapter 6: Raindrop Size Distribution ==============

1. Suppose it is raining. Assume the raindrop size distribution is described by the Marshall-Palmer Distribution.

(i) If the rain rate is $R = 2\ mm/h$, what percentage of the raindrops have a diameter larger than 1 mm?

(ii) Suppose the rain rate $R = 10\ mm/h$, what percentage of the raindrops have a diameter larger than 1 mm?

The fraction of raindrops larger than 1 mm (0.1 cm) can be obtained by integrating N(D) from 0.1 cm to infinity, divided by the integral of N(D) from 0 to infinity (i.e. normalizing by the total number of raindrops). Note that the integral of N(D) over all D does not equal one, but is designed to have units corresponding to the number of raindrops per m^3*, per mm interval (its an odd unit, in a way not consistent).*

M-P Question: (i) $\lambda = 35.4$*; should get 3 percent. (ii)* $\lambda = 25.3$*; should get 8 percent.*

2. Suppose that the rain rate is $R = 1$ mm/hour, but in one case all droplets have a radius $r = 0.1$ mm, while in the other case all droplets have a radius $r = 0.5$ mm. Assume that the terminal velocity of the raindrops is given by $u = k_3 r$, where $k_3 = 8 \times 10^3\ s^{-1}$.

(i) Find the precipitation water content L in each case (in units of mm^3 water per m^3 of air). The precipitation water content L is simply the volume of rainwater per volume of air. It is related to rain rate R and the velocity of the droplets u by $R = Lu$.

(ii) In each case, find the number of droplets per m^3 of air.

(iii) In each case, find the reflectivity Z of the air parcel for each case, where the unit volume is taken to be 1 m^3. (Hint : do not use the Marshall Palmer or observationally based $Z - R$ relationships.

============ Chapter 6: Graupel and Snow ==============

1. How does the size of a snowflake typically vary with temperature? Why?

Most snowflakes are aggregates of individual crystals. Aggregation occurs more readily at temperatures closer to 0 °C, where ice crystals are stickier (think snowballs). Wind shear can also rip snowflakes apart, so largest snowflakes will occur under calm conditions near 0 °C where the density of snowflakes is high (large snow rates), so snowflakes will experience more collisions. Under such conditions, can also expect to have large radar reflectivities, due to size and density of scatterers.

2. Do you think it's possible for ice crystals from a precipitating cloud to reach the surface without aggregation or riming? Does this happen? Explain.

I sometimes see snowflakes of individual hexagonal ice crystals on my window sill. Not that common though since would be tough for a small crystal to reach the surface without evaporating.

3. (i) In what season is graupel most commonly seen at the surface? Why?

Graupel requires larger updraft speeds, so some degree of convection. This is more common in summer. Most of mid-latitude summer rain was likely graupel at some point. However, graupel in summer usually melts into rain before reaching the surface. You do see graupel sometimes in winter, even though rarely have convective updraft speeds. I would answer "spring" or early summer, but I am not sure this question has an obvious answer. Would be regionally dependent.

(ii) What are two conditions which favour the formation of graupel?

higher updraft speeds, larger density of supercooled droplets (these conditions tend to go together actually).

4. A graupel particle is growing in a cloud with liquid cloud water content M. Assume that the cloud droplets have negligible fall velocity and that the updraft velocity of the cloud is much smaller than the graupel fall speed. The collision efficiency between the cloud droplets and graupel particle is 1. The graupel mass depends on graupel radius according to $m = aR^3$. The graupel fall speed depends on graupel radius according to $u(R) = k_3 R$. Derive an expression for the dependence of the graupel radius R on time.

5. Do you think it is possible that drizzle-sized raindrops can arise from the condensational growth of ice crystals? What evidence supports or rejects this possibility?

Individual ice crystals are observed to reach the surface. So if the 0 C height is above the ground, there is no reason why drizzle can't come from individual ice crystals (page 167). However, the atmosphere under the cloud would have to be close to saturation, since small raindrops evaporate very quickly as they fall.

============= **Chapter 6: Hail** ================

1. A hail particle is suspended in an updraft and is experiencing wet growth (i.e. its temperature is at 0 °C). Specify, and briefly discuss, three terms in the heat budget of the hail particle. Assume the temperature of the background atmosphere is -20 °C. Specify whether each terms would usually give rise to a heating or a cooling of the hail particle.

The main terms are:
(1) Freezing heating in the interior of the hail particle if the ice at the core is growing.
(2) During wet growth (water layer on surface of ice), the hail must be at 0 °C. But is in an updraft typically much colder (e.g. -20 C). Therefore the hail particle is losing heat by thermal diffusion.
(3) Supercooled droplets hit the hail from above or below. They are small and so would have the same temp as the updraft, e.g. -20 C, so would need to warmed for the hail particle to have a stable temp of 0 C (maintain wet growth).
(4) The water film on the outer surface of a hail particle will be evaporating since the water vapor pressure e in the updraft will be much less than $e_s(0C)$. So will be experiencing evaporative cooling.
A number of people referred to water droplets as "condensing" on the hail particle. However, we refer to water vapor as condensing. Condensation refers to a change of phase from gas to liquid. You could say the supercooled droplets accumulate on the hail particle, or collide and stick, even absorbed is better. But the water in the water droplets is already "condensed", so can't condense a second time.

2. A hail particle is suspended in an updraft and is experiencing wet growth (i.e. its temperature is at 0 °C). These questions concern the heat budget of the hail particle.

(i) What is the main source of heat to the growing hail particle?

(ii) What are two processes which would tend to reduce the temperature of the hail particle below 0 °C?

3. Growing hailstones tend to be warmer than their environment. Why?

============ Chapter 6: Melting Snowbanks ===============

1. Suppose a heavy rainstorm delivers 50 mm of rain at 10 °C. What is the maximum amount of snow on the ground (in kg/m^2) this amount of rain could melt (i.e. if all the rain cools to 0 °C)?

2. A saturated air parcel at 10 °C with a dry air mass $m_d = 2$ kg hits a snowbank. The snow is at 0 °C. While hitting the snowbank, the air parcel is chilled to 0 °C, and any water vapor in excess of e_s condenses on the snowbank and becomes water at 0 °C. Assume that all of the enthalpy lost by the air parcel (dry air and water vapor components) is used to melt a mass m_s of snow, which then becomes water at 0 °C. I would like you to solve for m_s using the following steps. Assume that the total atmospheric pressure is $p = 1000$ hPa. REPLACE BY VERSION IN ex.tex.

(i) What is the initial mass of water vapor in the saturated 2 kg air parcel at 10 °C?

For saturated parcels, $e = e_s(T)$. At 10 °C, $e_s = 12.2$ hPa. Since $p = p_d + e$, $p_d = 1000 - 12.2 = 987.8$ hPa. Use $w_v = \epsilon e/p_d = 0.00768$ to get the water vapor mass fraction. The mass of water vapor equals w_v times the mass of dry air. Here, $m_d = 2$ kg, so $m_v = 0.0154$ kg. Note: there is no need to find volumes or densities in this question. Trying to do so makes the question more complicated. Also, you should always use the ideal gas law for water vapor or dry air individually, but never together, i.e. use $e = \rho_v R_v T$ or $p_d = \rho_d R_d T$. Some of you used expressions like $p = \rho_d R_d T$, which is incorrect for a parcel with non-zero e.

(ii) What is the mass of water vapor that condenses?

The remaining water vapor pressure in the chilled air parcel is equal to e_s at 0 °C. This is given as 6.11 hPa in the formula sheet. Do exactly the same procedure as for part (i) to get the water vapor mass at 0 °C. You should get 0.0077 kg. Take the difference to find the mass of water vapor that must have condensed. You should get $m_{v,cond} = 0.00769$ kg.

(iv) Assume that there is no external heating, and that the process occurs at constant pressure. What is the mass m_s of snow that is melted?

See class notes.

3. I was speaking with my neighbor after she walked her dog. She said that rain would help get rid of the snowbanks, but what really gets rid of snow fast is fog. Why might snowbanks shrink faster under foggy conditions than rain?

In Halifax, very foggy conditions are associated with the transport of warm moist air from the Gulf Stream, which is chilled to its dew point as it crosses the Labrador current, and fog forms. When this warm moist air reaches Halifax, it would melt snowbanks because some of the remaining water vapor would condense on the surface of the cold snowbank, heating it by condensational heat release. This heat source would help melt the snow. Of course, sometimes you see foggy air being formed as wind blows warm moist air into a snowbank, so some condensation occurs in the atmosphere as well, but the part that does the melting would be the condensation on the snow. The main point would be that any kind of warm humid air hitting a snowbank would melt it very quickly. The presence of the fog would be mostly incidental, but usually in N.S. indicates the presence of warm, moist air.

========= Chapter 6: Hurricane Genesis and Distribution =======

1. During El Nino events, the frequency of hurricanes in the Atlantic Ocean is below normal. Explain.

did not do.

2. The climatological pattern of tropical cyclogenesis (hurricane formation) depends on three main factors. What are they?

in class notes.

3. Hurricane genesis differs from the genesis of many other types of storms, such as winter storms that arise from baroclinic instability? Explain briefly.

hurricanes need a trigger, some pre-existing vorticity anomaly.

4. Would hurricanes form on a non-rotating planet? Discuss why or why not. I will give credit for any physically reasonable argument.

Less likely since would lack f, so smaller source of vorticity to start with.

============ Chapter 6: Hurricane Intensity ==============

1. An increase in the intensity of a hurricane can be attributed to the existence of some positive feedback loops. Briefly discuss one of these.

did not do.

2. Specify, and briefly discuss, the two most important reasons why few hurricanes achieve their maximum theoretical intensity?

shear, or mixing of cold ocean water from below.

3. It continues to be difficult to forecast hurricane intensity. Give one reason for this difficulty.

Size of clouds smaller than grid box size in a forecast model, so have to make approximations on their behaviour; hard to know depth of ocean thermocline.

4. It is difficult for traditional weather forecast models to make accurate predictions of hurricane intensity. Specify and briefly discuss two specific challenges traditional forecast models encounter when trying to make hurricane intensity forecasts. These should be challenges that do not ordinarily give rise to errors in day to day forecasting.

Many people gave answers like chaos, which do present difficulty to hurricane forecasting, but I was looking for challenges which were unique to hurricane forecasting. These include:
(i) knowledge of thermocline depth along the hurricane track,
(ii) knowledge of heat/moisture transfer across ocean surface at high wind speeds,
(iii) knowledge of wind shear + RH in the mid-troposphere,
(iv) knowledge on how to model convective cloud mixing at the coarse grid resolution of numerical forecast models.

5. The growth rate of a hurricane will be sensitive to how it modifies its environment, and whether these changes in the environment increase or decrease the strength of a hurricane.

(i) Specify and very briefly discuss one way in which hurricanes change their environment in such a way as to increase their growth rate.

hurricanes increase the RH of their environment, so increase their intensification by diminishing the impact of entrained background on buoyancy loss (i.e. "environmental" or "out of cloud" air).

(ii) Specify and very briefly discuss one way in which hurricanes change their environment in such a way as to decrease their growth rate.

Cool the thermocline, or ocean mixed layer, by bringing up colder ocean water from below.

============ Chapter 6: Hurricane Structure ===============

1. (i) Suppose that the background large scale sea level pressure around a hurricane is 1000 hPa, and that the sea level pressure at the center of a hurricane is 900 hPa. Let z refer to height, and r refer to the radial distance from the center of the hurricane. In the $r - z$ plane, make a rough cross-sectional plot of how you would expect the following isobars to vary with z and r: 1000 hPa, 980 hPa, 960 hPa, 940 hPa, 920 hPa, 900 hPa, 800 hPa, 600 hPa, 400 hPa, and 200 hPa.

see class notes for warm core low, or hurricane.

(ii) With reference to your plot in (i), explain why the strongest winds of a hurricane are typically found in the bottom 1 - 2 km.

Aspects I was looking for: was $r = 0$ the place where the 900 hPa contour hit $z = 0$? At large z, was the 1000 hPa contour placed on the $z = 0$ axis? As general knowledge, in the background atmosphere, you should know that 200 hPa is roughly 12 km and 900 hPa is roughly 1 km. Were the relative height locations of these isobars roughly correct? At $z = 0$, did pressure decrease relatively continuously as you approached $r = 0$? Was there an awareness that hurricanes are warm core systems, so that the dp between two isobars in the lower troposphere tended to be larger than in the background atmosphere? Was there a dp/dr reversal in the upper troposphere, so that the air could get pushed out away from the hurricane? I did give full marks to diagrams that were not "perfect", but judged close enough, since I am looking for a correct qualitative picture.

The dp/dr at low levels that drives the inward pressure gradient acceleration is due to the downward slope of the isobars as r gets smaller. As you go up, the warmth at the center of a hurricane increases the vertical distance dz between isobars. This makes the isobar slope smaller, reduced dp/dr, and via cyclostrophic balance, decreases v.

2. (i) Show a schematic cross-section of the shape of pressure levels through the center of a hurricane. The vertical axis should be height above sea level, and extend from the surface to the tropopause. The horizontal axis should be the radial distance from the center of the hurricane, and extend some distance from the edge of the hurricane. Assume the 500 hPa pressure surface can be considered flat, both within the hurricane and in the background atmosphere. Indicate this pressure surface on your diagram.

(ii) On your diagram, indicate the direction of the pressure gradient acceleration near the surface, and the pressure gradient acceleration in the upper troposphere.

(iii) Starting from the center of a hurricane at 500 hPa, and going toward the background atmosphere, would you expect the vertical distance dz between adjacent pressure surfaces to increase or decrease?

Explain.

(iv) How does the hurricane change the geopotential height distance dZ between pressure levels in the mid-troposphere?

Heating due precipitation formation (condensation) pushes pressure surfaces apart at mid-levels, displacing pressure surfaces above 500 hPa up, and displacing pressure surface below 500 hPa down.

3. In the Northern Hemisphere, all hurricanes rotate counterclockwise (cyclonically). Briefly explain. A diagram may be helpful.

f is an important source of starting vorticity for a hurricane, and $f > 0$ in the NH, so rotation is counter clockwise. Hurricanes also usually require an additional vorticity from their "trigger", but this is also always positive.

4. The boundary layer of a hurricane extends from the surface to 2 km. The vertical velocity at the surface is zero, while $w = 1$ m/s at the top of the boundary layer. The vertical velocity increases linearly with height from the surface to the top of the boundary layer. The hurricane is at a latitude of 10 °, where $f = 2.53 \times 10^{-5}$ s−1.

(i) What is the value of the divergence in the boundary layer?

From dw/dz. (i) 0.0005

(ii) Suppose a parcel enters the hurricane boundary layer with $\xi = 0$ originally, and resides in the boundary layer for one hour. What is the value of the relative vorticity ξ of the air parcel at the end of 1 hour?

Should get exponential growth of relative vorticity if solve DE correctly.

============ Chapter 6: Hurricanes and Sea Level ==============

1. Hurricanes affect sea level in a number of ways. Specify and briefly discuss two different ways in which hurricanes modify the height of the ocean surface.

2. Hurricanes are often accompanied by a storm surge (increase in sea level). From a beach, why might you expect an increase in sea level as the eye of a hurricane approaches?

*Give result and set up : heuristic derivation of change in sea level height due to wind - set up carefully. go over $dp/dx = g\ dh/dx$ carefully detailed derivation of L*V2/H*

============ Chapter 6: Lightning ==============

1. A charge builds up on the surface of an ice crystal growing by vapor deposition. What is the sign of this charge and what is its origin?

in class notes

============ Chapter 6: Radar ==============

1. Suppose all raindrops have the same radius. What would be the change in reflectivity if the raindrops had double the radius, but the average water content (e.g. g rain/kg air) of the rain was kept constant. i.e., if Z_R is the initial reflectivity and Z_{2R} the final reflectivity, what is Z_{2R}/Z_R?

If the volume of water is constant, and you double the size of the drops, since each drop has 8 times as much volume, the number of drops must go down by 8 (volume goes as r cubed). However the reflectivity goes as r to the sixth power, so each drop will have 64 times more reflectivity. So 8 times more return.

2. Radars have difficulty seeing drizzle. Explain.

Don't see small droplets very well; return goes as diameter to the sixth power.

3. Vertical cross-sections of radar backscatter through precipitating clouds often show a bright band near 0 °C. Specify two physical processes that contribute to this bright band. (You may also give three reasons and I will count the two best explanations).

in class notes.

============ Chapter 6: Convection, General ==============

1. What are two physical processes that can affect the Convective Available Potential Energy (CAPE) at a location?

surface heat or moisture fluxes, radiative cooling aloft, cold advection aloft.

2. Draw a plot of how the average rainfall rate depends on sea surface temperature in the tropics (averaged over 30 °S - 30 °N).

flat at about 1.5 mm/day till an SST of 26 °C, then rapid increase, then rapid decline. See handout.

3. Draw a plot of the variation of the average rainfall rate with sea surface temperature in the tropics (averaged over 30 °S - 30 °N). Give a physical explanation for one important feature of the diagram.

4. In the tropics over the warm oceans, clouds which penetrate the boundary layer inversion and rise as high as 5 km are very common. However, most such clouds do not develop into deep convective clouds. Why not?

loss of buoyancy due to mixing.

============ Chapter 6: Tornadoes and Supercells ==============

1. A hodograph is a plot in which the y axis refers to the meridional wind and the x axis refers to the zonal wind, and on which is superimposed a vertical wind profile. Scientists have constructed composite hodographs by averaging over many background wind conditions when tornadic storms are present.

(i) Draw a typical composite hodograph for the central U.S., which reflects conditions in which tornadic supercell storms are present. The wind profile should extend from the surface wind to 200 hPa. Indicate the surface and 200 hPa background winds on the hodograph. Also show the typical average tornadic storm motion vector.

in text.

(ii) With reference to the surface wind you have shown in part (i), explain why this wind direction favors the development of thunderstorms.

warm moist air from the Gulf of Mexico.

2. Assume that a tornado is a Rankine vortex. It is in solid body rotation, where the tangential speed v is given by $v = v_0(r/r_0)$ for $r < r_0$, and $v = v_0(r_0/r)$ for $r > r_0$. Assume that the radial pressure gradient and the tangential speed v are related via cyclostrophic balance. Assume that the density ρ can be treated as a constant. Show that the pressure deficit at the center of the tornado, with respect to the background atmosphere, is given by $\delta p = \rho v_0^2$.

straightforward

3. (i) Assume that a supercell updraft is in solid body rotation with angular velocity $\omega = 0.02\ \mathrm{s}^{-1}$ out to a radius $r_0 = 500$ m. Estimate the pressure deficit δp at the center of the updraft compared to the background atmosphere. The density of the atmosphere is 1.25 kg/m^2. Express your answer in hPa.

In the text.

(ii) Assume that the angular velocity and pressure deficit given above characterize the updraft at 0.5 km. The angular velocity and pressure deficit under the updraft are both zero at the ground. Estimate the average vertical acceleration, due to the nonhydrostatic pressure gradient acceleration, on air parcels between the ground and 500 m.

Calculate the change in the pressure deficit, divide by 500 m, and divide by the parcel density.

4. The development of tornadic supercells is almost exclusively a phenomenon of the American mid-West. Specify two characteristics of the background atmosphere in this region that encourage supercell development.

strong shear, access to warm moist air from the Gulf of Mexico.

5. Within the updraft of a supercell, there is a nonhydrostatic pressure gradient force that accelerates air parcels upward. You may find that a diagram helps you answer these questions.

(i) How does the tangential velocity v within a supercell updraft typically vary with altitude, going upward away from the surface.

Tangential speed should increase in bottom few km, as escape frictional drag.

(ii) What is the approximate mathematical relationship between the tangential velocity v and the radial pressure gradient inside a supercell updraft?

asking for cyclostrophic balance.

(iii) Explain how this relationship, and the vertical variation of v, give rise to the nonhydrostatic vertical pressure gradient acceleration.

Pressure deficit with respect to background atmosphere at the center of a rotating updraft (as required to supply centripetal acceleration) must increase with height as tangential speed v increases. Therefore pressure at center of updraft decreases faster than what you would expect based on hydrostatic balance.

6. (i) What type of background wind environment favors the development of supercells?

Wind at low levels from the south, wind at upper levels from the west.

(ii) Why does this environment favor supercell development?

Provides vorticity which can be tilted into the vertical as parcels enter updrafts, plus air from the south needed to supply heat and moisture (maintain high CAPE).

3. What are the two main sources of vorticity to a supercell updraft?

convergence (stretching) and tilting

4. Suppose an updraft is situated in a background wind configuration in which the wind at low levels is westward (i.e. to the west), and the background wind at upper levels is eastward (i.e. to the east).

(i) What is the direction of the vorticity vector of this background flow? Explain briefly.

to the north using right hand rule.

(ii) Suppose a parcel approaches the updraft from the south. Assume tilting is the only source of relative vorticity in the updraft. Would the parcel in the updraft have positive or negative ξ?

positive. A useful analogy may be a person walking northward up a hill with an arrow protruding from his stomach. As he ascends the hill, the arrow will point up (see class notes).

(iii) Suppose a parcel approaches the updraft from the north. Assume tilting is the only source of relative vorticity in the updraft. Would the parcel in the updraft have positive or negative ξ?

negative

3. What are the two main sources of vorticity to a supercell updraft?

4. Suppose an updraft is situated in a background wind configuration in which the wind at low levels is westward (i.e. to the west), and the background wind at upper levels is eastward (i.e. to the east).

(i) What is the direction of the vorticity vector of this background flow? Explain briefly.

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(iii) Suppose a parcel approaches the updraft from the north. Assume tilting is the only source of relative vorticity in the updraft. Would the parcel in the updraft have positive or negative ξ?

=========== Chapter 6: Boundary Layer =================
=========== Wind profiles and Thermal Stratification =========

1. Assume that the mean wind $\overline{u}(z)$ in a boundary layer is in the positive x direction, and that the eddy induced frictional deceleration (or force) on the wind is constant in the boundary layer and zero above the boundary layer. Make a rough plot of how you would expect the turbulent vertical momentum flux $\overline{w'u'}$ to vary with altitude from the surface to just above the boundary layer.

If the eddy force is constant, then the eddy momentum flux is linear in height. It is also zero at the top of the BL. Momentum is transported down gradient (usually), so the eddy momentum flux must be down, or negative. So eddy momentum flux starts off being zero at the top of the BL and linearly increases toward the surface. Momentum is being removed from the BL and given to the surface (see handout).

2. You are making a forecast of surface wind, and have to decide whether to issue a wind warning or not. The horizontal wind in both cases is 40 m/s at $p = 900$ hPa. In one case, the surface temperature is 15 °C, while in the other case the surface temperature is 18 °C. If all other factors are equal, in which case would you expect the surface winds to be stronger? Explain.

In general, free tropospheric (i.e. above the BL) momentum is more easily transported to the surface under conditions of weaker stability, so would expect stronger surface winds when the surface temperature is warmer. In Nova Scotia for example, we tend to have weaker lower tropospheric stability in the fall when the oceans are warm relative to the atmosphere. This probably contributes to high surface wind events in our fall.

3. (i) Within the surface layer, show how the wind speed varies with height in the presence of (i) a very stable temperature profile, and (ii) an unstable stratification. Show the two wind speed profiles on the same plot.

didn't do.

(ii) Does this plot help explain any changes in surface wind speed at sunset? Explain.

didn't do.

4. Assume: $\overline{v}(z) = 0$, $\overline{\theta}(z) = 300 + 0.005 * z$ (z in m), and that $< T >= 295$ K. If the mean $\overline{u}$ wind is zero at the surface, estimate the threshold $\overline{u}$ value at 1 km in m/s that will just initiate turbulence.

Set $R_i = 0.25$.

5. (i) Turbulent Kinetic Energy (TKE) is generated within layers in which the atmosphere is thermally unstable (defined here as $\frac{d\theta}{dz} < 0$). Explain with a diagram.

Diagram showing rising parcels will have $B > 0$ *and sinking parcels will have* $B < 0$. *Important to show parcel* θ *conservation up and down.*

(ii) Turbulent Kinetic Energy (TKE) is consumed within layers in which the atmosphere is thermally stable (defined here as $\frac{d\theta}{dz} > 0$). Explain with a diagram. (Note that TKE is produced by shear $R_i < 0.25$ instability. Question is asking for a loss specific to stable layers. In general can be many sources and sinks of TKE.)

in class notes

2. Vertical turbulent mixing of a stably stratified atmosphere is said to consume (or extract) kinetic energy from the background flow. Explain why this is the case. Use a diagram to support your arguments.

============ Chapter 6: Boundary Layer ==============

5. Specify three *distinct* physical variables or processes that can affect the depth of a boundary layer. Indicate whether the boundary layer depth is increased or decreased by each variable or process.

in notes

6. What is a sink of turbulent kinetic energy in a convectively unstable boundary layer?

The best answer is dissipation. Could also say friction.

1. What are the two sinks of mechanically generated turbulent kinetic energy in a stable boundary layer (i.e. potential temperature increasing with altitude)?

loss of TKE to buoyancy, and to viscous dissipation

10. (i) An initially stable layer is completely mixed by mechanically induced turbulence. Show schematic profiles of the initial and final potential temperature profiles $\theta(z)$. Denote the top of the boundary layer in your diagram.

(ii) Show a schematic vertical profile of the turbulent eddy heat flux $\overline{w'\theta'}$ as a function of altitude, extending from the surface to some altitude above the top of the boundary layer. On your diagram, indicate the height interval where you would expect the divergence of the eddy heat flux to heat the background atmosphere, and the height interval where you expect the heat flux to cool the background atmosphere. Assume the eddy heat flux at the surface is equal to zero.

2. Vertical turbulent mixing of a stably stratified atmosphere is said to consume (or extract) kinetic energy from the background flow. Explain why this is the case. Use a diagram to support your arguments.

2. A convectively unstable boundary layer is growing into an overlying stable area. Plot how the vertical turbulent heat flux would typically vary with altitude from the surface to just above the top of the boundary layer. On the same plot, show how the local turbulent heating rate due to a divergence in the turbulent heat flux would vary with altitude. On both plots, indicate the following significant altitudes: the top of the boundary layer (z_i), the altitude at which the mean potential temperature starts increasing with altitude (z_{pos}), and the altitude at which the turbulent heat flux reaches its minimum value (z_{min}).

in handout.

3. Specify three physical variables or processes that can affect the depth of a boundary layer. Indicate whether the boundary layer depth is increased or decreased by each process.

in class notes.

4. The upper part of a boundary layer is being cooled at a constant rate by turbulent heat transport. The lower part of the boundary layer is being warmed at a constant rate by turbulent heat transport. Show a schematic plot of the turbulent heat flux $\overline{w'\theta'}$ as a function of altitude in the boundary layer.

Start from the top of the BL where the turbulent heat flux is zero. If the upper part of the BL is being cooled, the turbulent heat flux must be increasing with height. Therefore, from the top of the BL and going down, the turbulent heat flux must become progressively negative. Then must increase toward the surface where you have heating.

5. (i) Show a plot of the typical variation of inversion layer height z_i against time from sunrise to early afternoon. You can assume summer fair weather conditions over land, and that z_i is responding purely to the local solar forcing, rather than any changes in wind speed or temperature advection.

didn't do.

(ii) The rate of change of z_i shown in (i) is typically not constant over the course of the day. Why not?

didn't do.

6. There are various ways of defining the boundary layer. What are two characteristics of boundary layers that make them distinct from the free troposphere?

friction force non-zero, well mixed, turbulent heat fluxes, etc.

7. In large North American cities during the summer, there tends to be a strong positive correlation between elevated boundary layer ozone concentrations and temperature. What is the origin of this correlation?

didn't do, but high temperature events are usually associated with strong subsidence and low boundary layer heights, so pollution is capped near the ground.

8. Specify and briefly discuss the two physically distinct ways in which atmospheric angular momentum is transmitted from the atmosphere into the ground.

Looking for brief descriptions of viscous drag and form drag. Think of your car. Horizontal pressure gradient between front and back wind shields causes form drag. Turbulence all along surface of car causes viscous drag.

9. How would you expect the potential temperature and vertical heat flux to vary with height during the day above the ground in the presence of a thick forest canopy. Show vertical profiles of each, and indicate the height of the top of the canopy on your plots.

didn't do

10. How would you expect the Bowen ratio to vary with latitude over the oceans? Explain.

larger over warm oceans (see below).

11. (i) Define what is meant by open cell versus closed cell convection. A diagram may help.

didn't do

(ii) Briefly mention a factor which may favour one type of convection over the other.

didn't do

12. What are the two main sources of turbulent kinetic energy (TKE) to the boundary layer?

mechanically (wind shear) and convectively ($d\theta/dz < 0$) generated turbulence. Obstacles can also generate turbulence, e.g. wake induced turbulence behind an island.

13. At 1 km: $p = 900$ hPa, $T = 280$ K, $u = 10$ m/s, and $v = 0$ m/s. At 1.5 km: $p = 850$ hPa, $T = 275$ K, $v = 0$ m/s. What is the value of u at 1.5 km that will just initiate turbulence from an initially laminar flow?

Solve for θ at the two heights, and get $d\theta/dz$. Set $R_i = 0.25$ and solve for u at 1.5 km.

14. In class, we discussed a number of local and large scale factors which can influence the structure of the boundary layer. Mention two of these and discuss how and why these factors change the structure of the boundary layer.

in class notes

Bowen ratio typically larger over warm or cold areas of the ocean? Explain.

It's usually larger over warm areas of the ocean, since e_s and q_s are higher. The difference $q_s - q$ in general higher also.

given temp difference, and BL depth, find dynamic heat flux and temp change in BL.

7. The surface of a lake has a temperature of 0 °C. The air above the lake has a temperature of 10 °C, and a relative humidity $RH = 0.8$. The wind speed 10 m above the lake surface is 5 m/s. Assume that $C_H = C_E = 0.002$. The total atmospheric pressure is $p = 1000$ hPa. es at 0 C = 6.11 hPa es at 10 C = 12.3 hPa

(i) Estimate the kinematic moisture flux F_{water} at the surface of the lake.

Main issue was calculating specific humidity q. By far the easiest formula to use is $q = \epsilon e/p$. In the air, $e = RHe_s$, while $e = e_s = e_s(T_s)$ for the surface of the lake.

(ii) Would you expect the latent heat flux at the lake surface to be positive (upward) or negative (downward)? Explain.

Downward, since moisture flux is negative (downward). There is always some evaporation and condensation, but here the condensational flux exceeds the evaporative flux. The net condensational flux heats the lake surface (Water vapor molecules smash into the lake surface like asteroids as they fall into the potential well of the water surface, with their large kinetic energies distributed among the other water molecules after the collision, heating the water).

============ Chapter 6: Energy Budget ===============

1. (i) What are the two main sources of kinetic energy in the atmosphere?

Baroclinic and convective

(ii) Where and how is most of the kinetic energy in the atmosphere destroyed?

Most viscous dissipation (the ultimate sink of KE) occurs in the boundary layer.

2. Both the North Pacific and North Atlantic Ocean Basins have rotational "gyre" circulations. These circulations have an effect on the way the Atlantic and Pacific Oceans exchange energy with the atmosphere.

(i) In what parts of these circulations does there tend to be a net upward flux of energy into the atmosphere from the ocean (annually averaged)?

didn't do

(ii) In what parts of these circulations does there tend to be a net downward flux of energy from the atmosphere into the ocean (annually averaged)?

didn't do

(iii) In steady state, what term in the heat budget of the ocean mixed layer balances these net exchanges of heat with the atmosphere? Explain.

didn't do

3. For a typical mid-latitude location, make a plot of the seasonal variation in temperature 1 m below the surface and 3 m below the surface (over the course of a year). Indicate the month of year on the horizontal axis.

Figure is in text. Main points are that amplitude of seasonal temperature cycle decreases as go below the surface, and get increasing phase lag.