PHYS/OCEA 4411/5411 Atmospheric Dynamics I Questions

=========== **General** =======================

1. (i) What is a barotropic fluid? (4 points)
(ii) What part of the earth's atmosphere tends to most closely approximate a barotropic fluid? (4 points)

2. What is the main difference between a diagnostic and prognostic set of equations? Give an example of each type. (6 points)

3. What is an example of a flow with very small number Rossby number R_0, i.e. $R_0 \ll 1$? (2 points)

4. What are the two main sources of diabatic heating in the atmosphere? (8 points)
condensation and radiation

5. (i) What is the main source of θ in the atmosphere? (Another way of asking, is what is the main mechanism by which air parcels rise up through θ surfaces?) (2 points)

condensational heating. Some people gave answers like warm advection. Warm advection affects $\partial\theta/\partial t$, but not $d\theta/dt$.

(ii) What is the main sink of theta in the atmosphere? (2 points)

Radiative cooling from greenhouse gases. Could also say evaporative cooling from rainfall falling through unsaturated air, but this cooling is smaller.

6. An air parcel encounters a baroclinic zone where there is a rapid decrease in temperature on a pressure surface, in the direction of its motion. You can assume that the diabatic heating Q of the parcel is small.

(i) Would you expect this parcel to stay on the same pressure surface, sink to a higher pressure surface, or rise to a lower pressure surface? Explain. (2 points)

(ii) Would you expect this air parcel to produce clouds and (possibly) precipitation? Explain. (2 points)

8. How does the period of the buoyancy oscillation of an incompressible fluid vary as it becomes less stably stratified and approaches a state of constant density? Buoyancy oscillations refer to the oscillations that result when parcels are displaced vertically in a stably stratified fluid. (6 points)

The restoring force goes to zero as the fluid approaches constant density. The period of the oscillation therefore goes to infinity. This is analagous to the situation where a spring becomes progressively weaker, or a guitar string progressively more loose, and is no longer able to sustain an oscillation. In the atmosphere, this occurs when θ becomes constant with height.

9. There is a layer of warm water with $\rho_w = 1000 kg/m^3$ above a layer of cold water with $\rho_c = 1001 kg/m^3$. If a parcel of cold water is displaced into the layer of warm water, what is its buoyancy acceleration? Assume that water is incompressible. (8 points)

Treat water as an incompressible fluid. Then the densities of a water parcel originating from one of the layers will be constant if it is displaced vertically. In this case, the acceleration equals the normalized density difference multiplied by g: $g(\rho_w - \rho_c)/\rho_c$. Note that you can not apply the formulas used for an atmosphere (assumed ideal gas) to an incompressible fluid.

4. Suppose an ocean made of vinegar is connected to an ocean made of oil by a narrow channel. The two oceans have the same temperature. In what direction would the surface current flow between

the two oceans? Explain your choice. (8 points)
This is a question about baroclinicity. Oil is lighter. To lower the gravitational potential energy, the denser fluid (vinegar) will flow under the lighter one. So the surface current will flow from oil to vinegar. Similar to cold air on one side of room and warm on other side. For example, Mediterranean is warm and salty and Black Sea is cold and fresh. Turns out the effect of salt on density is more important than temperature, so the surface current in the Bosphorus flows from the Black Sea to the Mediterranean (and so the strong currents and colder water makes it harder to swim across).

========== **Temperature Advection** ========================

1. There is a large scale temperature gradient at the surface in which the temperature decreases northward by 0.5 ^{o}C every 100 km. The wind is blowing to the northeast at 10 m/s. A thermometer Moving with the wind records a temperature decrease of -2 ^{o}C/day. What is the the rate of temperature change an observer on the ground would measure (in ^{o}C/day)?

It is best to express the velocity as a vector $\mathbf{U} = (7.07, 7.07)$ *m/s, and the T gradient as a vector* $\bigtriangledown \mathbf{T} = (0, 5 \times 10^{-6}) K/m$. *You are given* dT/dt. *Use the formula, making the dot product of the vectors, to find* $\partial T/\partial t$. *Then convert from K/s to K/day. Keep in mind the temperature gradient vector points from cold to warm, so points south, so the dot product with the wind vector is negative. Always draw a diagram and decide what the sign of the advection term should be, as a check.*

2. (i) The wind is blowing to the northeast at 5 m/s. What is the horizontal wind vector $\mathbf{U} = (u, v)$? (10 points)

(ii) The temperature is colder toward the north at 1 C per 400 km, and warmer toward the east at 1 C per 600 km. Suppose that the temperature is 0 C at the origin. What is T(x,y)? (6 points)

(ii) What is the temperature gradient vector $\nabla \mathbf{T}$? (6 points)

(iii) The sun is heating the air mass so that the rate of temperature change as measured by a thermometer traveling with the wind is 2 C/day. Calculate the local rate of temperature change in this situation (i.e. the rate of temperature change calculated by a stationary observor.) (12 points)

3. Atmospheric scientists often separate the temperature variability at the surface over the course of a day into advective (transport) and diabatic (e.g. solar) components. In what season would you expect the relative influence of advection on the daily temperature variability to be largest? Explain. (8 points)

1. (i) The wind is blowing to the northeast at 5 m/s. What is the horizontal wind vector $\mathbf{V} = (u, v)$? (too easy) (8 points)

(ii) The temperature is colder toward the east at 1 ^{o}C per 400 km. It is constant in the north-south (meridional) direction. What is $T(x, y)$, assuming the temperature is zero at the origin? (8 points) (Could make T changing in two directions. A good exercise: some students have difficulty writing down a scalar function in two spatial dimensions.)

(iii) What is the temperature gradient vector $\nabla \mathbf{T}$? (6 points)

(iv) The sun is heating the air mass so that the rate of temperature change as measured by a thermometer traveling with the wind is 2 °C/day. Calculate the local rate of temperature change in this situation (i.e. the rate of temperature change calculated by a stationary observer.) (8 points)

=========== **Scale Analysis** ========================

1. Synoptic scale motions in mid-latitudes ($\phi \sim 45^o$) have horizontal velocities $u \sim v \sim 10$ m/s, vertical velocity $w \sim 1$ cm/s, horizontal length scale $L \sim 10^6$ m, vertical length scale $H \sim 10^4$ m, and horizontal pressure fluctuations $\delta p/\rho \sim 10^3$ m^2/s^2. Perform a scale analysis of the du/dt primitive equation to determine the order of magnitude of each of the terms in this equation. Note that $f = 2\Omega \sin\phi \sim 10^{-4}s^{-1}$ at $\phi = 45^o$. Ignore the friction term. (20 points)

=========== **Coriolis** ========================

1. Prove :

$$-2\mathbf{\Omega_{vert}} \times \mathbf{U} = (fv, -fu)$$

$\mathbf{\Omega_{vert}}$ is the projection of the rotation vector of the earth $\mathbf{\Omega}$ on to the local vertical. (20 points)

2. Suppose the earth were a rotating cylinder instead of a sphere.

(i) Would air parcels experience a horizontal Coriolis acceleration? Explain. (6 points)

(ii) What would be the main effect of the rotation? (2 points)

============= **Gravity** ===================

1. (i) We derived in class an expression for the acceleration in u (du/dt) due to a non-zero meridional velocity v on a rotating planet. In point form, try to outline the main 3 - 4 steps along the way. There is no need to show equations, but you may show a diagram and refer to mathematical variables. (12 points)

A non-zero meridional velocity $v = \delta y/\delta t$ changes the distance R from the earth's axis of rotation. To express this effect, you need a geometric relationships between δy and δR. The change δR changes the moment of inertia of the parcel. To conserve angular momentum, the angular velocity ω of the parcel must change. The total angular velocity is $\omega = \Omega + u/R$, so u must change.

(ii) What is the sign of du/dt for $v > 0$? Explain. (6 points)

$v > 0$ means the parcel is getting closer to the earth's axis of rotation, so the moment of inertia goes down, which means the total angular velocity ω goes up, which means u goes up.

2. In general, is the geometric distance dz between two geopotential surfaces larger at the equator or the poles? Explain. (6 points)

Effective gravity is weaker at the equator so have to travel a larger dz to do the same work against effective gravity.

3. You have a mass of 50 kg are driving down the highway at the equator in an eastward direction at 20 m/s. Estimate the difference in weight from when you are stationary (also at the equator). Express your answer in N. (15 points)

The easiest way is to calculate the $2\Omega u \cos\phi$ term of the dw/dt primitive equation with $\phi = 0$. Could also calculate the change in the centrifugal acceleration, $(\Omega + \omega)^2\mathbf{R} - \Omega^2\mathbf{R}$, where $\omega = u/a$ is the additional angular velocity due to the eastward velocity (positive since in the same direction as the *rotation of the earth), and a is the earth radius.* **R** *is the position vector to the location at the equator, which you can just replace by a.*

6. (Almost identical to question above) A person with a mass of 50 kg is driving down the highway at the equator in an eastward direction at 25 m/s. There are two corrections to his weight that can be attributed to his non-zero zonal velocity. Calculate these two corrections. Express your two answers in N.

The easiest way to answer this question is to pick out the two terms of the vertical momentum equation below and multiply each acceleration by g.

$$\frac{dw}{dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u \cos\phi + F_{rz}$$

At the equator, the sum of these two terms is also equal to the centrifugal acceleration difference

$$\Omega^2 a - (\Omega + \frac{u}{a})^2 a$$

The question asked for the modification in weight due to a non-zero u, so answers discussing the non-spherical shape of the earth, or the rotation of the earth itself, did not directly address the question.

5. What are the main two reasons a person weighs less at the equator than the poles?

A person at the equator has a smaller "true" gravitational acceleration than a person at the poles because he is about 21 km further from the earth's center of gravity. Secondly, the centrifugal acceleration is opposite to the gravitational at equator, and is proportional to R (distance from the rotation axis).

6. When traveling down the highway in a car, is your effective weight larger traveling east or west? Explain.

7. Suppose an earth day had 12 hours instead of 24 hours. How would this change the shape of the geopotential height surfaces on the earth? Briefly explain. A diagram may be helpful.

8. In general, is the geometric distance dz between two geopotential surfaces larger at the equator or the poles? Explain.
3. Does the geometric distance dz between two geopotential height surfaces usually increase or decrease in going from the equator to the North Pole? Explain. (6 points)

================== Primitive Equations =====================

1. In order to solve for the time evolution of the six variables (u, v, w, p, ρ, T) in a dry atmosphere, we need six equations. What are these six governing equations? I am looking only for the names of these equations. There is no need to write them out. (18 points)

The governing equations are the three momentum equations, plus conservation of mass and energy, and the equation of state. The set of governing equations includes all the physics you have decided to include in a physical problem, and allows you in principle to predict the evolution of the system. It should therefore include a prognostic equation for each of the physical variables, or a diagnostic equation enabling you to solve for the time evolution of a variable given knowledge of the other variables. Governing equations usually invoke basic conservation laws. For example, the primitive equation for du/dt helps solve for the time evolution of u taking into account the four forces of gravity, *viscosity, pressure gradient, and Coriolis. It is important to distinguish between a governing equation, and particular expressions, or versions, of the four forces. Conservation of energy is a prognostic equation for temperature, and conservation of mass a prognostic equation for density. The equation of state is also a governing equation, despite being diagnostic.*

========== Dissipational Heating/Turbulence ==============

1. Suppose all shears in a fluid are constant, i.e. u depends linearly on z, etc.

(i) Is the viscous acceleration zero? (3 points)

(ii) Is the viscous heating zero? (3 points)

2. Under what conditions would viscous dissipational heating be most likely to be a significant source of heating in the atmosphere? Explain. (6 points)

3. (i) What is a turbulence closure? (2 points)

(ii) Why are they neccessary? (2 points)

(iii) Give an example of a turbulence closure. (2 points)

4. Is dissipational heating usually important in the atmosphere? Why or why not? (6 points)

======== Rotational Flow in a Cup/Cylinder =============

4. (i) You are stirring a cup of tea with a spoon. Pick reasonable values for horizontal speed U and horizontal size L. Estimate the magnitude of the centripetal acceleration of the fluid parcels in the tea. (8 points)
(ii) Using the most reasonable acceleration balance for this type of situation, and your answer to (i), estimate the difference at the bottom of the cup between the pressure at the center and the sides. Assume the density of the tea equals the density of water. (10 points) (Note: this is a reasonable estimate, not an exact calculation.)
(iii) Using your answer to (ii), estimate the height difference between the top of the tea at the center and at the sides. (10 points)
(iv) Assume there is friction at the bottom of the cup. Show, using a diagram, the secondary circulation associated with this frictional deceleration. (6 points) (Say that should show vertical cross-section.)

1. An incompressible fluid, with constant density ρ, is exhibiting solid body rotation in a circular cup. You can assume the existence of a force balance in which the pressure gradient acceleration is exactly equal to the centripetal acceleration. Let $h(r)$ refer to the radial variation of the height of the surface of the fluid in the cup. Let H refer to the height of the fluid at the center of the cup, so that $h(0) = H$. Ignore atmospheric pressure and friction.

(i) Assume that the period of rotation of the fluid in the cup is T. This is the time required for a fluid parcel to execute one complete rotation. Express the tangential speed $v(r)$ in terms of T and r.

$$v(r) = \frac{2\pi r}{T}$$

(ii) The pressure in the fluid at any depth is equal to the hydrostatic pressure of the fluid above it.

Let $p(r)$ refer to the pressure at the bottom of the cup as a function of radius. Express $p(r)$ as a function of $h(r)$, g, and ρ.

In hydrostatic balance, the pressure at any point in a fluid is equal to the weight per unit area of the fluid above it. Let δA refer to a little area at the bottom of the cup. The volume of a column with this area is equal to $\delta A\, h(r)$. Since you can ignore the atmosphere, the pressure at the bottom of the cup is equal to

$$p(r) = \frac{\delta A\, h(r) \rho g}{\delta A} = h(r)\rho g$$

(iii) Given cyclostrophic balance as discussed above, derive an expression for $dh(r)/dr$ in terms of g, T, and r. (4 points)

In cyclostrophic balance the inward centripital acceleration is provided by the pressure gradient acceleration, here due to the curvature of the fluid at the top surface.

$$\frac{v(r)^2}{r} = \frac{1}{\rho}\frac{dp(r)}{dr}$$

Use the relation in (iii).

$$\frac{v(r)^2}{r} = g\frac{dh(r)}{dr}$$

$$\frac{dh(r)}{dr} = \frac{v(r)^2}{rg}$$

(iv) Integrate this relationship to solve for $h(r)$ in terms of r, T, g, and H. (4 points)

Substitute the relationship from (i).

$$\frac{dh(r)}{dr} = \frac{4\pi^2 r^2}{rgT^2} = \frac{4\pi^2 r}{gT^2}$$

$$dh(r) = \frac{4\pi^2}{gT^2} r\,dr$$

Integrating and using the condition $h(0) = H$ gives.

$$h(r) = H + \frac{2\pi^2}{gT^2} r^2$$

2. A cup of tea is undergoing rigid body rotation at 1 revolution per second. Let f = 1.0E-04 /s. The cup has a radius of 5 cm.

(i) What is the relative vorticity ξ of the flow? (6 points)

(ii) What is the absolute vorticity η of the flow? (4 points)

(iii) Estimate the Rossby number of this circulation. (6 points)

(iv) Would you expect this flow to be closest to the inertial, cyclostrophic, or geostrophic balance limits? Explain. (4 points)

3. A cylindrical tank of water is spinning with angular velocity of rotation ω. Let H be the depth of the water at the center of the tank. Assume that water is incompressible with density ρ_0. The atmospheric pressure is p_0.

(i) Express the pressure gradient acceleration $PGA(r)$ in terms of ω and r, where r refers to the distance from the center of the tank. You can assume that the flow is frictionless, and that f = 0.

(ii) Let $h(r)$ be the depth of the water in the tank as a function of r. Note that $h(0) = H$. Express the pressure at the bottom of the tank in terms of $h(r)$, ρ_0, g, and p_0, assuming that the water is in hydrostatic balance.

(iii) Derive an expression for $h(r)$ in terms of r, g, and ω.

4. When stirring a tea cup, the tea leaves tend to cluster near the bottom center of the cup. Explain. Make sure to discuss the physical origins of the relevant circulation. Use a diagram to show the direction of the circulation.

5. Show that for rigid body rotation, the relative vorticity is equal to twice the angular velocity, $\xi = 2\omega$.

========== Hydrostatic Equation ========================

1. Derive the hydrostatic approximation $dp/dz = -\rho g$. Note the main assumption used in the derivation. (25 points)

In the notes. The main assumption is that parcel has no vertical acceleration.

2. A wave is traveling along the ocean surface. A starfish at the bottom of the ocean is interested in the deviation of the pressure from the hydrostatic pressure.

(i) During what parts of the wave is the pressure at the bottom of the ocean less than the hydrostatic pressure? Explain qualitatively. (By "parts of the wave", I mean with respect to the peaks or the troughs, or to the midpoints between a peak and a trough where the amplitude is zero. A diagram may help.) (6 points)

(ii) During what parts of the wave is the pressure at the bottom of the ocean larger than hydrostatic pressure? Explain qualitatively. (6 points)

It is important to distinguish hydrostatic pressure from the deviation from hydrostatic pressure. The hydrostatic pressure itself is largest under the peaks (where the water depth is larger) and smallest under the troughs (where the water depth is smaller). However, $dw/dt < 0$ under the peaks, so the fluid is in partial free fall at locations where the fluid is above the equilibrium depth. The pressure in the fluid is therefore less than the hydrostatic pressure under the peaks. Conversely, the fluid is being accelerated upward under the troughs, so the bottom pressure is larger than the hydrostatic pressure. It may help to imagine yourself as a fluid parcel and bouncing up and down on a giant bungee cord, and ask yourself when you will feel in free fall (above equilibrium height), and where you will feel heavier and stretching the cord more (below equilibrium height).

3. The figure below shows a wave traveling along the surface of the ocean. Locations A', B', and C' are on the ocean floor directly under A, B, and C. Ignore the role of atmospheric pressure in the following questions. Where appropriate, simply circle the best answer.

What the figure shows: a sinusoidal wave on the ocean surface moving to the right. A is at the peak of the wave, B is at the next place to the right where the amplitude is zero, and C is at the next place to the right where the amplitude reaches its largest negative value.

(i) Rank A', B', and C' in terms of the magnitude of the hydrostatic pressure at each location, largest to smallest. (1 point)
The hydrostatic pressure is largest where the depth of the water above is largest (i.e. A' under the peak), and smallest where the depth of the water above is smallest (i.e. C' under the trough).

(ii) In which location would expect the largest column averaged downward velocity: between A and A', between B and B', or between C and C'? (1 point)
The velocity should be closest to zero under the troughs and peaks (where water parcels reach their extremum points). Since the wave is moving to the right, the upward velocity should be largest near B. So the answer would be: between B and B'.

(iii) In which location would expect the largest column averaged convergence: between A and A', between B and B', or between C and C'? (1 point)
You need the largest mass convergence where the height of the water surface is increasing most rapidly. This would be between B and B'.

(iv) In which location would expect the largest column averaged upward acceleration: between A and A', between B and B', or between C and C'? (1 point)
Between C and C' where the velocity is zero, water parcels have reached their lowest point (like the bottom of a mass bouncing up and down on a spring).

(v) In which location would expect the largest column averaged downward acceleration: between A and A', between B and B', or between C and C'? (1 point)
Between A and A' where the velocity is zero, water parcels have reached their highest point (like the top of a mass bouncing up and down on a spring).

(vi) In which location would you expect the pressure at the bottom to exceed hydrostatic pressure by the largest amount: A', B', or C'? (1 point)
You need larger surface pressure (relative to hydrostatic) at the bottom at the place where the water column is accelerating upward most rapidly, i.e. C'.

(vii) In which location would you expect the pressure at the bottom to be smaller than hydrostatic pressure by the largest amount: A', B', or C'? (1 point)
You would expect smaller surface pressure at the bottom (relative to hydrostatic) at the place where the water column is in partial free fall most rapidly, i.e. A'.

======== **Heating and Layer Thickness** ===========

2. (i) What is the mass of a column of air with area A = 1 m^2 between 1000 hPa and 900 hPa? (6 points)
(ii) The air within the layer is being heated at a rate of $\dot{Q} = 0.1$ J/kg s. Within the layer, the pressure velocity is zero (i.e. $\omega = 0$). What is the rate of temperature change dT/dt within the layer? Express your answer in K/day. (10 points)
(iii) Suppose the the geopotential height at 1000 hPa is fixed at Z = 0 (i.e. MSL). How much would the geopotential height of the 900 hPa surface be increased if this heating rate is sustained for one day (in m)? (12 points)
9. Assume that the 800 hPa surface is flat, i.e. located at a fixed geopotential height of 2 km above

the surface. There is a temperature gradient under the 800 hPa surface such that the mass weighted virtual temperature at 300 K at one end, and 280 K at the other end. These warm and cold regions are separated by a distance of 100 km.
(i) Estimate the surface pressure at the cold end.
(ii) Estimate the direction and magnitude of the pressure gradient acceleration at the surface between the warm and cold ends. Use any reasonable means to estimate an average density under the 800 hPa surface. Use any reasonable means to estimate an average density under the 800 hPa surface.

========= Energy Equation ============

3. (i) Suppose that air is flowing over a hill such that the adiabatic flow is in steady state (i.e. $\partial p/\partial t = 0$), there is no friction, and that the kinetic energy of the flow is dominated by the speed in the x direction (i.e. u). Assume that the temperature of the air is constant as the air flows over the hill and that the density $\rho = 1$ kg/m^3 is also constant. The upstream flow starts off at MSL with $Z = 0$, $u = 5$ m/s, and $p = 1000$ hPa. At the top of the hill, $Z = 50$ m, and $u = 20$ m/s. What is the pressure at the top of the hill? (12 points)
(ii) What would you have expected the pressure at the top of the hill to be based on assuming hydrostatic balance? (8 points)

========= Calculating Vertical Velocity - Divergence ============

1. The surface pressure at the center of a low is 960 hPa, and is decreasing at 20 hPa/day. Assume that the horizontal and vertical wind at the surface at the center of the low is zero.

(i) What are ω and $\partial p/\partial t$ at the surface at the center of the low? (4 points)
(ii) Between 960 hPa and 800 hPa, there is a uniform convergence of $\nabla \cdot \vec{V_h} = -0.2$/day. What is ω at 800 hPa? (12 points)

(iii) Assume adiabatic conditions and that $T = 260$ K at 800 hPa. What is the rate of temperature change dT/dt of an air parcel at 800 hPa? (10 points)
1. An air parcel is rising at a constant vertical speed w. The parcel is being heated by radiation at $\dot{Q} = 0.1$ W/kg. What is the vertical speed w (in m/s) of the parcel, if the temperature of the parcel is constant and ? You can assume that the atmosphere is in hydrostatic balance and that $p(z)$ is fixed.

Use the following most convenient form of the thermodynamic equation.

$$c_p \frac{dT}{dt} - \alpha \frac{dp}{dt} = \dot{Q}$$

You are given that the temperature is constant, $\alpha = 1/\rho$, and use $\omega = dp/dt$.

$$-\omega = \dot{Q}\rho$$

In these conditions, can use $\omega = -\rho g w$.

$$\rho g w = \dot{Q}\rho$$

$$w = \frac{\dot{Q}}{g}$$

2. An air parcel at the surface is at rest (including the vertical velocity $w = 0$). However the vertical pressure velocity ω of the parcel is negative. Give a physical explanation of how this could occur. (8 points)

$\omega < 0$, $dp/dt < 0$ is true by definition (didn't give many points for just this.) From hydrostatic balance, the pressure at any height is equal to the overhead column mass per unit area (times g). Anything that reduces the overhead column mass will reduce p: e.g. divergent outflow, precipitation.

3. The horizontal wind speed (u, v) is defined at 4 points about the origin. The point 100 km to the east of the origin has $(u, v) = (8, 9)$. The point 100 km to the west of the origin has $(u, v) = (10, 0)$. The point 100 km to the north of the origin has $(u, v) = (8, 1)$. The point 100 km to the south of the origin has $(u, v) = (12, 0)$. All units are in m/s. Assume $f = 1 \times 10^{-4}$ s^{-1}.

(i) What is the relative vorticity ξ at the origin? (4 points)

Use $\xi = dv/dx - du/dy$. Here $dx = dy = 200$ km, $dv = 9$ m/s - 0 m/s (using the two points to the east and west for which dx is non zero), and $du = 8$ m/s - 12 m/s = - 4 m/s (using the two points to the north and south for which dy is non-zero). Should get 6.5E-5 /s. In these questions, it is helpful to draw the vectors.

(ii) What is the divergence at the origin? (4 points)

divergence is $du/dx + dv/dy$. $du = 8$ m/s - 10 m/s = - 2 m/s. $dv = 1$ m/s - 0 m/s = 1 m/s. Should get -5E-06 /s.

(iii) Estimate the rate of change of the absolute vorticity of the air parcel at the origin, following its motion (i.e. $d\eta/dt$), due to the divergence term. (4 points)

Take your answer in (ii) and multiply by the total vorticity.

(iv) Suppose the horizontal winds given above can be considered the averaged winds between 800 hPa and 600 hPa, and that the vertical pressure velocity at 800 hPa is 0.1 Pa/sec. What is the vertical pressure velocity at 600 hPa in hPa/day? (4 points)

Use the kinematic expression for ω, which really just comes from the continuity expression (conservation of mass). Keep in mind this expression can be used between any two pressure levels. The divergence from (ii) should be negative, i.e. a convergent influx of air between 800 hPa and 600 hPa. This could represent an upward vertical velocity increasing with altitude (air being entrained into an upraft for example), or a downward velocity that increased toward the surface. Both of these cases require a source of mass. In this case, the velocity is downward (positive omega) at 800 hpa, so the velocity at 600 hPa should be less downward, i.e. smaller omega. You get omega(600 hPa) = omega(800 hpa) - 0.1 hPa/s = 0.1 hPa/s - 0.1 hPa/s = 0. Remember to convert hPa to Pa. People often make a sign error in the pressure interval.

4. The figure below shows contours of constant geopotential height (solid lines) and contours of constant temperature (dashed lines) on the 700 hPa pressure surface. At locations A and B, indicate the following:

What the figure looks like: There is a trough in GPHT (with larger GPHT to the south). There are lines of constant temperature going straight east west with colder temperatures to the north. Point A is at a downstream side of the trough. Point B is at the upstream side of the trough.
(i) The direction of the temperature gradient vector $\nabla \vec{T}$, (2 points)

Points straight south (toward warmer air at both locations). Gradient vectors point uphill from *low to high.*

(ii) The direction of the pressure gradient acceleration. (4 points)

The PGA points "downhill" to lower geopotential height on a pressure surface.

(iii) The direction of the geostrophic wind $\vec{V_g}$, (4 points)

parallel to geopotential height lines on a pressure surface with the PGA pointing left.

(iv) Specify whether you have warm or cold geostrophic temperature advection at A and B. (2 points)

The temperature advection is $-\vec{V_h} \cdot \nabla T$. *So warm advection at A and cold advection at B.*

(v) Specify whether you would expect up or down motion at A and B. Explain using a formula. (4 points)

Use $\omega \approx (-\nabla T \cdot \vec{V_h})/-\sigma_p$. *Because* $\sigma_p > 0$, *warm advection implies* $\omega < 0$, *so ascent. Unfortunately, the expression on the formula sheet did not include the* T *here, so marked generously.*

5. The surface pressure at the center of a low is 960 hPa. The surface pressure is deepening at 20 hPa/day. The horizontal and vertical wind at the surface at the center of the low is zero.

(i) What is ω at the surface at the center of the low? (2 points)

If the horizontal and vertical wind are zero, ω *and* $\partial p/\partial t$ *are the same. For any variable, the total and local derivatives are the same if there is no vertical or horizontal advection.*

(ii) What is $\partial p/\partial t$ at the surface at the center of the low? (2 points)

This is the rate of change of p at a fixed position. It is given as -20 hPa/day.

(iii) Between 960 hPa and 800 hPa, there is a uniform convergence of $\nabla \cdot \vec{V_h} = -0.2$/day. What is ω at 800 hPa? (12 points)

Use the equation of continuity to solve for ω_{800}.

$$\nabla \cdot \vec{V_h} = -\frac{\partial \omega}{\partial p} = -\frac{\omega_{800} - \omega_{960}}{p_{800} - p_{960}}$$

You should get -52 hPa/day, with -20 hPa/day coming from ω *at the surface, and an additional -32 hPa/day coming from the convergence. Note that a convergence will make* ω *more negative (higher contribute to upward motion). Be careful with the signs.*

(iv) Assume that $\partial p/\partial t = 0$ on the 800 hPa surface and that the horizontal wind is zero. The temperature is 280 K. What is the vertical velocity w at 800 hPa? PROBLEM HERE since partial derivs evaluated at fixed height usually. (8 points)

Under these conditions, it is appropriate to use $\omega \approx -\rho g w$ *to get* w *(see the notes for more detail on this derivation). Use* T *and* p *to solve for the density* ρ. *Convert* ω *to Pa/s if you haven't already. You should get 6.15 mm/s.*

(v) Assume the motion of the parcel at 800 hPa at the center of the low is adiabatic. What is the rate of temperature change dT/dt of an air parcel at 800 hPa following its motion? (8 points)

It is easiest to use the thermodynamic equation:

$$c_p \frac{dT}{dt} - \alpha \frac{dp}{dt} = \dot{Q}$$

Set $\dot{Q} = 0$*, solve for the density* ρ *and use to get* α*, and use* $\omega = dp/dt$ *from (iii). You get,*

$$\frac{dT}{dt} = \frac{\omega}{c_p \rho}.$$

Use $\omega = -52\ hPa/day = -0.060\ Pa/s$ *and* $\rho = 0.996\ kg/m^3$*, to get a cooling rate of* 6.02×10^{-5} *k/s, or -5.2 K/day.*

6. What is the divergence of the following vector field: $\mathbf{U}(x, y) = x\,\mathbf{i} + y\,\mathbf{j}$?

7. An air parcel is sitting at the surface, and does not move. The initial pressure at the top of the air parcel is $p_t = 900$ hPa. The pressure at the bottom of the air parcel is fixed at $p_b = 1000$ hPa. The air parcel is exposed to a divergence $\nabla \cdot \mathbf{V_h} = -0.2$ /day. The air parcel has a uniform density of $\rho = 1$ kg/m^3.

(i) What is the pressure velocity ω_t at the top of the air parcel (in hPa/day)?

(ii) Assuming adiabatic conditions, what is the rate of temperature change at the top of the box?

============ **Conservation of Angular Momentum** ============

1. An air parcel at the equator with zonal wind speed $u = 0$ is displaced to 15 $^\circ$N. If there are no zonal pressure torques exerted on the air parcel during this displacement, what is the zonal velocity of the air parcel at 15 $^\circ$N? (20 points)

Use conservation of angular momentum, $L = m\,\omega\,R^2$*, where* ω *is the total angular velcity, i.e.* $\omega = \Omega + u/R$*, where* Ω *is the angular velocity of the earth,* u *is the zonal wind speed, and* R *is the distance from the axis of rotation. At the equator* R *= Re, and you are given* u *= 0. You can calculate R at 15 N, and use L(0 N) = L(15 N) to solve for* u *at 15 N.*

============== **Kinetic Energy Generation** ==============

1. The actual wind is directed 30 degrees to the left of the geostrophic wind. If the geostrophic wind speed is 20 m/s, what is the rate of change of wind speed (i.e. solve for dV/dt). Let f = 1.0E-04. Assume the geostrophic wind is pointing south.

2. (i) Briefly discuss the main energy source of mid-latitude synoptic storms. A diagram may be helpful. (6 points)

(ii) Why are these storms more common in the winter hemisphere than the summer hemisphere? (4 points)

============== **Gradient Wind Balance** ==============

1. Let $|\mathbf{PGA}|$, $|\mathbf{Co}|$, and $|\mathbf{Ce}|$ refer to the magnitudes of the pressure gradient, coriolis, and centripetal accelerations. Let V and V_g refer to the magnitudes of the gradient and geostrophic wind speeds, respectively.

(i) For a regular low in gradient wind balance, which is larger: $|\mathbf{PGA}|$ or $|\mathbf{Co}|$? Explain. A diagram may help. (3 points)

(ii) For a regular high in gradient wind balance, which is larger: $|\mathbf{PGA}|$ or $|\mathbf{Co}|$? Explain. A diagram may help. (3 points)

(iii) For a regular low, which is larger: V or V_g? Explain. A diagram or equation may help. (2 points)

(iv) For a regular high, which is larger: V or V_g? Explain. A diagram or equation may help. (2 points)

2. At some location in the Northern Hemisphere, the local geostrophic wind is directed toward the west (i.e. an "easterly" wind) and has a speed of 20 m/s. The actual wind is directed 20 ^{o} to the south of the geostrophic wind, and also has a speed of 20 m/s. Assume $f = 1.0 \times 10^{-4}$.

(i) What are the two horizontal components of the geopotential gradient vector $\nabla_{\mathbf{p}}\phi$? (16 points)

$ug = -20\ m/s$ and $v_g = 0$. Use $u_g = -\frac{1}{f}\frac{\partial\phi}{\partial y}$ to solve for $\frac{\partial\phi}{\partial y}$. Should also say $\frac{\partial\phi}{\partial x} = 0$.

(ii) What is the rate of change of speed of the air parcel? (14 points)

Easiest way is to use natural coordinates: $\frac{dV}{dt} = -\frac{\partial\phi}{\partial s}$. The key is to relate $\frac{\partial\phi}{\partial s}$ and $\frac{\partial\phi}{\partial y}$. Use the geometric relation, $\delta s\, sin(20) = -\delta y$. Negative here since δy is in the negative direction.

(iii) What is the rate of change of kinetic energy of the air parcel? (6 points)

I should have given the mass. But $KE = 0.5mV^2$, so $d\,KE/dt = mV\,dV/dt$.

============ **Pressure Gradient Acceleration** ==========

5. A bathtub is 2 m long. The depth of the water is 1 cm deeper at one end than the other. Estimate the magnitude of the pressure gradient acceleration of the water in the bathtub. In what direction does it point? The density of water is 1000 kg/m^3. (10 points) (This is a bit easy. Maybe estimate sloshing frequency using dimensional arguments? From PGA = m/s2 = L/T2 = 0.045 m/s2, and L = 2 m, get T = sqrt(L/A) = 6.7 s.)

2. At a latitude $\phi = 35^o$, the pressure at 5 km is $p = 500$ hPa. At a latitude $\phi = 55^o$, the pressure at 5 km is $p = 450$ hPa. You can assume that the density $\rho = 0.50$ kg/m^3 is constant on the 5 km height surface, that the pressure gradient between the two latitudes is constant, and that $f = 1 \times 10^{-4}$ s^{-1}.

(i) What is the distance between the two latitude circles? (4 points) (A bit easy.)
(ii) What is mean pressure gradient acceleration between the two points? In what direction does this acceleration point? (8 points)
(iii) What is the geostrophic wind $\mathbf{V_g}$ at $\phi = 45^o$? (8 points) (Becomes a bit easy when have answer to (ii). Main problem is that some students do not use the component form, but the more cumbersome cross product form for the geostrophic wind.)

1. The surface pressure is constant at 1000 hPa. At point A, the average temperature in the 1000 - 900 hPa layer is 300 K. At point B, the average temperature in the 1000 - 900 hPa layer is 280 K. Point A is 1000 km to the west of point B. The average 1000 - 900 hPa layer temperature varies linearly between Point A and point B. Assume $f = 1 \times 10^{-4}$ s^{-1}.

(i) What is the pressure gradient acceleration on the 900 hPa surface at some point halfway between A and B? Does it point in the direction of A or B? (6 points)

Use the Thickness relationship to find the Z at A and B.

$$\Delta Z = \frac{R < T >}{g_0} ln(p_1/p_2)$$

Maybe in future: should break this into parts and explicitly ask for Z at the two locations.
Then convert the Z values above to geopotential. The pressure gradient acceleration vector is equal to minus the geopotential gradient. So you just take the geopotential difference between the two locations and divide by the distance, and express as a vector.

$$\mathbf{PGA} = -\nabla_{\mathbf{p}}\phi$$

(ii) What is the magnitude and direction of the geostrophic wind vector on the 900 hPa surface halfway between A and B.

It is easiest to use the scalar forms.

$$u_g = -\frac{1}{f}\frac{\partial \phi}{\partial y}$$

$$v_g = \frac{1}{f}\frac{\partial \phi}{\partial x}$$

Here, the geopotential varies only in the x direction, so $u_g = 0$, and v_g is easilly calculated from (i).

2. Assume that the surface pressure is 1000 hPa everywhere. At the equator, the average temperature of the 1000 - 500 hPa layer is 285 K. At 45 ^{o} N, the average temperature of the 1000 - 500 hPa layer temperature is 260 K.

(i) Calculate the 500 hPa geopotential height Z at the equator.

(ii) Calculate the 500 hPa geopotential height Z at 45 ^{o} N.

(iii) Estimate the direction and magnitude of the average pressure gradient acceleration between the equator and 45 ^{o} N on the 500 hPa surface.

Use the expression for PGA on a pressure surface: Pressure Gradient Acceleration : $\frac{\mathbf{F}}{m} = -\bigtriangledown_p \Phi$. *Here, $\bigtriangledown_p \Phi = (\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y})$, and $\frac{\partial \phi}{\partial x} = 0$. Use the change in Z from earlier to get the change in ϕ. δy is equal to the circumferance of the earth divided by 8. An important point is that, because you are asked to find the PGA on a pressure surface, you must use an expression for the PGA valid on a pressure surface.*

(iv) If the pressure gradient acceleration on the 500 hPa surface is constant between the equator and 45 ^{o} N, and the zonal pressure gradient acceleration is zero, how would you expect the magnitude V_g of the geostrophic wind to vary with latitude between the equator and 45 ^{o} N? Explain.

If PGA = CA, PGA is constant, then CA is constant with latitude. Since f increases with latitude, V_g must decrease.

3. An air parcel has a horizontal velocity on a pressure surface of $\mathbf{V} = (10, 5)$ in units of m/s. The geopotential gradient vector on this pressure surface is $\nabla_{\mathbf{p}}\phi = (0, 0.01)$ m/s^2. What is the rate of change in speed of the air parcel dV/dt?

Many people interpreted this question as asking for the vector acceleration $d\mathbf{V}/dt$. However the vector acceleration and dV/dt are quite different. For example, in solid body rotation the change in speed *of each fluid parcel is zero but there is a non-zero vector (here centripetal) acceleration toward the rotation axis. Notice also that the Coriolois acceleration is always directed at right angles to the velocity vector, does no work on, and therefore has no effect on the speed of an air parcel, so can be ignored in this question. The rate of speed of an air parcel is determined by the component of the PGA in the direction of* $\mathbf{V}$*. You can use the PGA vector (here* $-\nabla_{\mathbf{p}}\phi$*) and dot it with a unit vector in the direction of* $\mathbf{V}$ *(this would be* $\mathbf{V}/|\mathbf{V}|$*) to find* dV/dt*, or use.*

$$\text{Natural coordinates}: \frac{dV}{dt} = -\frac{\partial \phi}{\partial s}$$

4. Assume that geopotential Φ (or Z) surfaces are locally horizontal in the x direction. Assume also that isobaric (constant pressure) surfaces are not parallel to geopotential surfaces in the x direction, i.e. constant p surfaces have a slope in the x direction. As discussed on the web page, this implies that there is a non-zero pressure gradient acceleration in the x direction. In this case, show that

$$\frac{1}{\rho}\Big(\frac{\partial p}{\partial x}\Big)_Z = \Big(\frac{\partial \Phi}{\partial x}\Big)_p.$$

I.e. that the pressure gradient acceleration can be expressed as proportional to the pressure gradient on a height surface, or to the geopotential gradient on a pressure surface. (20 points)

5. A bathtub is 2 m long. The depth of the water is 1 cm deeper at one end than the other. Estimate the magnitude of the pressure gradient acceleration of the water in the bathtub. In what direction does it point? The density of water is 1000 kg/m^3.

================== **Circulation** ================

1. The surface pressure over both the land and ocean is 1000 hPa. The average 1000-900 hPa temperature over the ocean is 280 K. The average 1000-900 hPa temperature over the land is 290 K. You can assume $f = 0$.

(i) Estimate the rate of change of circulation dC/dt of a rectangle which goes between the land and ocean with the bottom edge at 1000 hPa and top edge at 900 hPa. (10 points)
(ii) Let $< v >$ refer to the mean tangential velocity along a rectangle where one end goes from 1000-900 hPa over the ocean, while the other end goes from 1000-900 hPa over the land, with the two ends being 20 km apart. Suppose $< v >= 0$ at $t = 0$. Estimate $< v >$ after 10 minutes. (14 points)
(iii) What is a source of inaccuracy in this calculation of d¡v¿/dt? (4 points)
(iv) At what time of year are sea breeze circulations strongest in Nova Scotia. Give two distinct reasons. (6 points)

1. The surface pressure over both the land and ocean is 1000 hPa. The average 1000-900 hPa temperature over the ocean is 280 K. The average 1000-900 hPa temperature over the land is 290 K. You can assume $f = 0$.

(i) Estimate the rate of change of circulation dC/dt.

(ii) Let $< v >$ refer to the mean tangential velocity along a rectangle where one end goes from 1000-900 hPa over the ocean, while the other end goes from 1000-900 hPa over the land, with the two ends being 20 km apart. Suppose $< v >= 0$ at $t = 0$. Estimate $< v >$ after 10 minutes.

(iii) Briefly discuss one source of inaccuracy in this calculation.

============= **Vorticity** ======================

1. (i) Start from the isobaric vorticity equation. Identify, as specifically as possible, the term in this equation that gives rise to the latitudinal restoring force for Rossby waves. (Hint: what I am looking for is not a term written in the formula sheet, but the part of one of these terms that provides the restoring force itself).
(ii) Show mathematically (and using a diagram) how this term tends to prevent air parcels from deviating too far (for both northward and southward deviations) from some initial starting latitude.

2. Which side of a westerly (i.e. eastward) jet in the North Hemisphere has larger relative vorticity: the equatorward side or the poleward side? Explain. (4 points)

3. Using a diagram, explain how a vertical velocity w which changes along the x axis, in combination with a meridional velocity v which varies in the vertical direction, can change (increase or decrease) the vertical component of the absolute vorticity. Make sure that you discuss the sign of the resulting vorticity tendency in relation to the signs of the w and v dervivatives you have selected. (15 points)

This question can be answered directly from the notes. I did not give full marks unless you explained how differential vertical advection of v by the gradient in w gave rise to a horizontal shear in v, which created a vertical component of vorticity.

=============== **Potential Vorticity** =================

2. There is an eastward (westerly) flow toward a mountain range, which is oriented north-south. The flow is adiabatic and frictionless. The 300 hPa pressure surface has a constant potential temperature of 340 K. The potential temperature at the ground (including the top of the mountain) is constant at 280 K. Assume that the bottom of an air parcel heading toward the mountain range has $\theta = 280$ K while the top is at $\theta = 340$ K. The initial latitude of the air parcel is 45 $^\circ$N, where $f = 1 \times 10^{-4}$ s^{-1}. Before striking the mountain, the westerly flow increases to the north at a rate of 5 m/s per 100 km. The surface pressure away from the mountain is $p_s = 1000$ hPa, while the pressure at the top of the mountain ridge is $p = 400$ hPa. You can assume that u and v are independent of height.

(i) What is the initial relative vorticity of the air upstream of the mountain range? (6 points)
(ii) What is the initial potential vorticity of the air upstream of the mountain range? (8 points)
(iii) Suppose that the air parcel crosses the mountain range at a latitude of 55 $^\circ$. What is the relative vorticity of the air parcel as it crosses the mountain? (16 points)
f at 55 = 1.2E-04 (a) -5E-5 (b) 4.3E-7 (c) -4.8E-06

1. A cylindrical column of air at 30 $^\circ$N expands isentropically (adiabatically) and without friction from a radius of of 100 km to a radius of 200 km. The column of air is initially at rest (not moving and not rotating). As it expands, it starts to undergo rigid body rotation.

(i) If the initial pressure thickness of the column is dp, what is the pressure thickness of the column after the isentropic expansion? (4 points)
The mass of the parcel can be defined as $\delta M = Adz * \rho$, where A is the area, dz the height, and ρ the density. Use the hydrostatic relationship $dp/dz = -\rho g$ for dz. Then $\delta M = -Adp/g$. The minus sign can be dropped here. It would be defined as the bottom pressure minus the top pressure, which is negative, and two negatives make a positive. If the initial and final masses are equal, then $A_i dp_i = A_f dp_f$. Since the final area is four times larger, the pressure difference dp must go down by four.

(ii) What is the relative vorticity of the column of air after the expansion? (6 points)

Use PV conservation. Since the difference in PT between the top and bottom of the parcel is constant, $\eta_i/dp_i = \eta_f/dp_f$, where i refers to initial and f refers to final. Using $dp_f/dp_i = 0.25$, the total vorticity must decrease by a factor of 4. Since you are given f in the beginning, and assume this does not change since at rest, you can find the final relative vorticity.

(iii) What is the mean tangential velocity at the perimeter of the column after the expansion? (4 points)

One way to do this is to say that vorticity is equal to circulation of a loop divided by area (technically only valid as area of loop goes to zero, but OK here if relative vorticity constant inside the are parcel.) In this case, relative vorticity $\xi = 2\pi R < v > /\pi R^2$, or $< v >= \xi R/2$. Another approach is to assume that the parcel is in rigid body motion. In this case $\xi = 2\omega$ where ω is the angular rotation rate of the parcel. Since $< v >= R\omega$, you get the same answer for $< v >$.

2. There is a eastward (westerly) flow toward a mountain range, which is oriented north-south. The flow is adiabatic and frictionless. The 200 hPa pressure surface has a constant potential temperature of 340 K. The potential temperature at the ground (including the top of the mountain) is constant at 280 K. The initial latitude of the air parcel is 45 ^{o} N. Before striking the mountain, the westerly flow increases to the north at a rate of 5 m/s per 100 km. The surface pressure away from the mountain is $p_s = 1000$ hPa, while the pressure at the top of the mountain ridge is $p = 400$ hPa. You can assume that the atmospheric stability is independent of height.

(i) What is the initial relative vorticity of the air upstream of the mountain range? (2 points)

(ii) What is the initial potential vorticity of the air upstream of the mountain range? (2 points)

(iii) As the flow approaches the mountain, would you expect the air to be deflected to the north, or to the south. Explain. (Note: the potential temperature at 200 hPa is assumed constant; this is not the "normal" meridional deflection). (2 points)

(iv) Suppose that the relative vorticity of the air parcel when it crosses the mountain range is zero. At what latitude does the parcel cross the mountain range? (4 points)

3. The difference in potential temperature between the top and bottom potential temperature surfaces of a tiny air parcel is $\delta\theta$. The air parcel is moving adiabatically through the atmosphere in the absence of friction. Let δC be the circulation of the air parcel. It was shown in notes that Kelvin's Circulation theorem implies that the $\delta C + f\delta A =$ *constant* for the air parcel. Assume that the mass of the parcel is constant, that the parcel is in hydrostatic balance, and that there is no heating within the parcel (i.e. the motion is adiabatic).Prove that this also implies that the potential vorticity PV of the air parcel, defined as $PV = (\xi_\theta + f)(-g\frac{\partial\theta}{\partial p})$, is also constant as the air parcel is advected through the atmosphere. ξ_θ refers to the relative vorticity, as calculated on a potential temperature surface.

This question is taken from the notes. It is the second part of the PV proof. Using the relationship between circulation and vorticity, $\delta C + f\delta A = \xi_\theta\delta A + f\delta A$. Express the area of the parcel in terms of the mass using $\delta M = \delta A\,\delta z\,\rho$, and use the hydrostatic relationship for δz. This gives $\delta M = -\delta A dp/g$, or $\delta A = -\delta M\, g/\delta p$. Using this, we have $-\delta M\, g/\delta p(\xi_\theta + f)$ as the new conserved quantity. Dropping the mass, then $-g/\delta p(\xi_\theta + f)$ is the conserved quantity. Since the parcel motion is adiabatic the $\delta\theta$ of the parcel is conserved, so $-g(\delta\theta/\delta p)(\xi_\theta + f)$ is conserved.

6. An air parcel is traveling eastward into a region where the stability is higher. Would you expect this parcel to curve toward the north, or the south? Explain.

7. Suppose there is a heat source that peaks in the mid-troposphere, and goes to zero near the surface and in the upper troposphere. Give a brief description of how you would expect this heat source to change the potential vorticity PV at various levels in the atmosphere.

8. What are the two main conditions required for conservation of Ertels potential vorticity?

The two main assumptions under which PV is conserved are adiabatic flow (no radiative or condensational heating/cooling), and no frictional or viscous forces. If you said conservation of mass, I gave full credit, since conservation of mass is invoked in the derivation of PV. However, the only way this is violated in practice is due to rainfall and associated removal of condensate mass from an air parcel, and in this case, the condensational heating would be a much bigger source of the PV change.

9. (i) An air parcel with zero relative vorticity ($\xi = 0$) and a latitude of 15° has a potential temperature $\theta = 300$ K at its lower surface of $p = 1000$ hPa, and a potential temperature $\theta = 310$ K at its upper surface of $p = 900$ hPa. What is the potential vorticity PV of the air parcel?

Use the definition of PV.

$$PV = (\xi_\theta + f)\left(-g\frac{\partial\theta}{\partial p}\right)$$

Should get $PV = 3.78 \times 10^{-07}\, Km^2/kg \cdot s$

(ii) The air parcel is exposed to a fixed convergence of 4/day, i.e., $\nabla \cdot \mathbf{V_h} = -4$ /day. The pressure velocity ω of the lower surface is zero (i.e. $\omega_{1000hPa} = 0$). What is the pressure velocity ω_{900} at the upper 900 hPa surface of the air parcel? Express your answer in hPa/day.

$$Use\ Continuity\ equation\ in\ pressure\ coordinates: \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$$

Should get $\omega_{900} = -400$ *hPa/day.*

(iii) The constant convergence between 1000 and 900 hPa is maintained for 1 day. Assume that the lower surface of the parcel continues to be fixed at 1000 hPa. What is the pressure of the upper surface at the end of a day (in hPa).

Given answer to (ii), upper surface goes from 900 hPa to 500 hPa in one day.

(iv) Assume that heating rate is zero within the air parcel and that all frictional effects can be ignored. What is the absolute (total) vorticity η of the parcel at the end of the first day?

PV should be conserved under these conditions. Use the answer to (i), and the new reduced stability with the parcel extending from 1000 hPa to 500 hPa. Under PV conservation, a reduction in stability must be accompanied by an increase in absolute vorticity η. *Should get* $\eta = 1.9 \times 10^{-4} s^{-1}$.

(v) Assume that the the latitude of the parcel does not change. What is the relative vorticity ξ of the air parcel at the end of a day?

Use the definition of the absolute vorticity $\eta = \xi + f$, *the value of* η *from (iv), and the previous value of* f. *Should get* $\xi = 1.51E \times 10^{-4} s^{-1}$.

(vi) Assume that the parcel is part of a hurricane in which the flow near the surface can be characterized as rigid body rotation (i.e. the tangential speed increases linearly with distance from the rotation axis $V(r) = \Omega r$, with Ω the angular velocity, and r the radius from the rotation axis). What is the angular velocity Ω of the air parcel about the rotation axis?

For rigid body rotation, $\Omega = 2\xi$. This is unfortunate, but the Ω here is not the rotation rate of the earth. Isn't really a good symbol to use. Use the value of ξ from (v). Should get $\Omega = 7.56 \times 10^{-5} s^{-1}$.

(vii) Assume also that the parcel is 200 km from the center of the hurricane. What is the tangential speed V of the air parcel (in m/s)? (Notes: It would be interesting to plot the dependence of the wind speed on the initial latitude of the parcel, to help show why hurricanes have difficulty generating rotation close to the equator. The planetary vorticity or f is too small, and this reduces PV. It is also true that, in order for hurricane genesis to occur, it is almost always required that the initial near surface absolute vorticity be larger than f, i.e. that the lower troposphere have some pre-existing positive relative vorticity to help get things started. Hurricanes usually evolve from some pre-existing initial finite low level vortex; they cannot bring themselves into existence from a constant background, relative vorticity free, initial state.)(8 points)

Use $V(r) = \Omega r$, with Ω from (vi). Should get 15.1 m/s.

==== Large Scale Shape of Pressure, Potential Temperature Surfaces ==

4. Assume that the surface pressure is 1000 hPa everywhere. At the equator, the average temperature of the 1000 - 500 hPa layer is 285 K. At 45 ^{o} N, the average temperature of the 1000 - 500 hPa layer temperature is 260 K.
(i) What is the 500 hPa geopotential height Z at the equator? (6 points)
(ii) What is the 500 hPa geopotential height Z at 45 ^{o} N? (6 points)
(iii) What is the direction and mean magnitude of the pressure gradient acceleration between the equator and 45 ^{o} N on the 500 hPa surface? (10 points)
(iv) If the pressure gradient acceleration on the 500 hPa surface is constant between the equator and 45 ^{o} N, and pressure gradient acceleration in the zonal (east-west) direction is zero, how would you expect the magnitude V_g of the geostrophic wind to vary with latitude between the equator and 45 ^{o} N? Explain. (4 points)

1. The geopotential height of the 500 hPa surface at the equator in January is roughly the same as July. Show a rough plot of how you would expect the height of the 500 hPa pressure surface to vary with latitude in going from the equator to the North Pole. Draw a curve for January, and one for July. The horizontal axis should be latitude with the equator at the left hand side. The vertical axis should be geopotential height. DRAW ON SAME PLOT. (4 points)

2. Make a rough plot of how you would expect the 500 hPa pressure surface to vary with altitude from the South Pole to the North Pole. Assume that it is January.

3. Make a rough plot of how you would expect a typical potential temperature surface (e.g. 330 K) to vary with altitude from the South Pole to the North Pole. Assume that it is January. (2 points)

4. Let the x axis refer to latitude between the equator and 90 ^{o} N. Let the vertical axis refer to height between 0 and 16 km. The average height of the 100 hpa surface is roughly 16 km at all latitudes, and so can be considered roughly "flat" with respect to a height surface. Assume that the surface pressure is 1000 hPa at all latitudes, and that it is winter in the Northern Hemisphere.

(i) Show very roughly how you would expect the heights of the 800 hPa, 600 hPa, 400 hPa, 300 hPa, and 200 hPa pressure levels to vary with latitude. (8 points)

(ii) Show the typical location of the mid-latitude jet on your figure. With respect to your figure, explain the increase in the geostrophic wind speed with height below the jet maximum, and decrease with height above the jet maximum. (8 points)

5. (i) Assume the surface pressure is constant in going from the equator to the pole. Would the geopotential height Z of the 500 hPa surface typically increase or decrease in going from the equator to the pole? Explain with reference to the most relevant equation. Would the rate of decrease be larger in the winter or summer? Explain. (12 points)

Looking for a discussion of the Thickness Equation. The stronger temperature gradient would give rise to a stronger decrease in geopotential height in the winter.

$$\Delta Z = \frac{R < T >}{g_0} ln(p_1/p_2)$$

(ii) Assume there is no temperature variation in the zonal direction (i.e. along longitude circles). Using a diagram, show the typical direction and relative magnitudes of the pressure gradient and Coriolis accelerations at a mid-latitude location on the 500 hPa surface. On the same diagram, indicate the typical direction of the geostrophic wind vector $\mathbf{V_g}$. (12 points)

If there is no change with temperature (also geopotential height) in the zonal direction, the PGA will point directly north. The actual wind is usually close to the geostrophic wind in mid-latitudes, so the CA will point roughly south. The geostrophic wind will be directed eastward.

============= **Boundary Layer** ==================

1. The figure below shows a parcel located on a geopotential height surface with pressure isobars as indicated. At the location of the parcel, there is a three way balance between the pressure gradient, frictional, and Coriolis acclerations (assume Northern Hemisphere). In this situation, draw the likely direction of the velocity vector, and the direction and relative magnitudes of the three acceleration vectors. (8 points)

2. What are two important factors which affect the depth of the Boundary Layer? (2 points)

3. Within a turbulent boundary layer, there is a downward flux of turbulent zonal momentum toward the surface. Supose you had an instrument which could measure the perturbations u' in zonal wind and w' in vertical velocity, at high time resolution.

(i) Would you expect the perturbations u' and w' to be positively or negatively correlated? Explain. (2 points)

(ii) Suppose that turbulence is exerting a eastward force on the mean flow in the boundary layer (i.e. $d\overline{u}/dt < 0$), and that this force is independent of depth in the boundary layer. Also assume that the flow at top of the BL is laminar. Show a schematic profile of how you would expect $\overline{u'w'}$ to vary with altitude in the boundary layer. Use height as your vertical coordinate. Indicate the surface, and the top of the boundary layer. (2 points)

4. (i) Using a diagram, show the direction of the 3 main forces on an air parcel in a turbulent boundary layer. Indicate lines of constant pressure, and the velocity vector of the air parcel. You can assume that the net force on the air parcel is zero. (6 points)

(ii) In the presence of turbulent friction, why is it always neccessary, from an energetic standpoint, for the wind vector of an air parcel to have a component toward low pressure? (2 points)

5. Suppose that the boundary layer is statically stable $d\theta/dz > 0$.

(i) Would you expect BPL to be positive or negative? Explain. A diagram may help. (2 points)

(ii) Suppose the boundary layer is turbulent, (i.e. TKE > 0). What is the likely source of the turbulence? (2 points)

(iii) What are the likely sink(s) of TKE? (2 points)

(iv) Would you expect the turbulent vertical heat flux to be upward or downward? Explain. (2 points)

6. (i) Under what circumstances are mixed layer boundary layers likely to occur? (2 points)

(ii) Make a plot of the vertical structure of a mixed layer boundary layer, showing and specifying the three layers, and the vertical variation of the mean wind $\overline{u}$ and $\overline{\theta}$ in the three layers. (6 points)

(iii) How do the vertical flux of zonal momentum $\overline{u'w'}$, and the frictional force vary with height in the mixed layer. Explain. (4 points)
7. The air parcel below is undergoing counterclockwise rotation about a low in the Northern Hemisphere. The lines below correspond to pressure isobars. There is a small amount of friction, which is affecting the velocity vector of the air parcel. The relative magnitudes of the acceleration vectors you draw should be realistic.

(i) Indicate the most likely velocity vector of the air parcel.
See notes.
(ii) Indicate the direction of the pressure gradient acceleration.
See notes.
(iii) Indicate the direction of the Coriolis acceleration. (1 point)
See notes.
(iv) Indicate the direction of the frictional acceleration. (1 point)
See notes.
(v) What is the secondary circulation that develops in a spinning cup of tea in response to friction at the bottom of the cup? A diagram would help.

Inward at the bottom, upward at the center, outward at the top, downward at the sides.
(vi) How do the vertical and horizontal motions associated with this secondary circulation contribute to a decrease in the rotation of the tea?

You can say friction at the bottom decreases angular momentum, allowing fluid parcels to flow in toward the center at the bottom. This low momentum fluid is transported upward at the center. As this low momentum fluid spreads outward to the sides of the cup at the top, it causes the fluid to start spinning down. So the destruction of angular momentum at the bottom is distributed throughout the cup by the secondary circulation.

Near the bottom of the cup, friction introduces a force opposing the tangential velocity. This reduces the tangential speed. The inward pressure gradient acceleration now exceeds the inward acceleration required for circular motion (i.e. cyclostrophic balance). Parcels of tea will start being deflected toward the center. This frictionally induced convergence at the bottom leads to upward motion at the center of the cup near the bottom (as indicated by the congregation of tea leaves). The upward flux at *the center is balanced by downward motion along the sides of the cup. This is the secondary vertical and horizontal circulation superimposed on top of the faster circular flow, or primary circulation.*

============== **Geostrophic Wind** ================

1. The geostrophic wind at 600 hPa is 20 m/s directed to the north. The mean temperature of the 600 - 500 hPa layer decreases to the north at a rate of 1 K per 100 km. Let f = 1.0E-04.

(i) What is the thermal wind of the 600 - 500 hPa layer? Give (u, v) components. (14 points)

(ii) What is the average geostrophic wind vector $\mathbf{Vg} = (u_g, v_g)$ of the 600 - 500 hPa layer? (10 points)

(iii) What is the temperature gradient vector of the layer ∇T? (5 points)

(iv) What is the rate of geostrophic temperature advection in the layer? Express in K/day. (5 points)

2. (i) At 30 ^{o}N, the mean temperature between the surface (1000 hPa) and 500 hPa is is 285 K. What is the geopotential height Z at 500 hPa? (8 points)

Use thickness expression to get 5780 m.

(ii) At 50 ^{o}N, the mean temperature between the surface (1000 hPa) and 500 hPa is is 270 K. What is the geopotential height Z at 500 hPa? (8 points)

Use thickness expression to get 5475 m.

(iii) What is the horizontal distance between 30 N and 50 N? (i.e. in the meridional or north-south direction) (4 points)

(iv) Assume that the geopotential height Z varies linearly with latitude on the 500 hPa surface between 30 ^{o}N and 50 ^{o}N. What is the direction and magnitude of the pressure gradient acceleration between these two latitude circles? (8 points)

(v) What is the direction of the geostrophic wind between 30 ^{o}N and 50 ^{o}N. There is no change in Z in the east-west direction. (4 points)
"eastward" or "westerly" or "points east".

(vi) What is the speed of the geostrophic wind $\mathbf{V_g}$ at 40 ^{o}N on the 500 hPa surface? (8 points)
14 m/s.

3. The geopotential height Z of the 500 hPa surface decreases northward at a rate of 80 m for every 100 km.

(i) What is the magnitude and direction of the pressure gradient acceleration on the 500 hPa pressure surface? (15 points)

Given GP gradient on a pressure surface, so easiest to use PGA $= -\bigtriangledown_p \Phi$. PGA will be directed north, since $\partial\Phi/\partial y < 0$ and $\partial\Phi/\partial x = 0$.

(ii) If the latitude is 45 ^{o}N, what is the direction and magnitude of the geostrophic wind? (15 points)

Set PGA + CA = 0. Note that, in general, both are vectors, but here PGA is only along the y axis, so CA will also be along the y (north-south) axis but directed south. Since CA acts to the right of V_g, $V_g = (u_g, 0)$.

4. What are two conditions in which the geostrophic wind tends to be a poor approximation to the actual horizontal winds? (Note: you do not have to explain why the geostrophic approximation fails.) (10 points)

in BL where friction is larger, in tropics where f is small, for dynamical motions where the Rossby number is not small (most general explanations: covers others), in locations where the isobar curvature is large.

5. Suppose it is winter and there is an average meridional temperature difference of 30 $^\circ$C between 20 $^\circ$N and 60 $^\circ$N. This is a temperature difference on pressure surfaces that is sustained between 900 hPa and 300 hPa, and gives rise to a uniform meridional temperature gradient on these pressure surfaces. If the zonal geostrophic wind at 900 hPa at 45 $^\circ$N is zero, what is the zonal geostrophic wind u_g at 45 $^\circ$N on the 300 hPa surface? (25 points)

Calculate dT/dy and use the thermal wind equation for uT. Here dy is the distance between 20 N and 60 N, i.e. 1/9 of the circumference of the earth. Should get about 21 m/s.

6. An observer on the 500 hPa pressure surface feels a wind from the south west. There is a temperature gradient on this pressure surface, with warm air to the south, and cold air to the north. In what direction would you expect the geostrophic wind above the observer to rotate: clockwise or counter clockwise? Explain your choice with a diagram and any relevant equations. (4 points)

7. (i) The geopotential height of the 500 hPa surface decreases from 5.0 km to 4.92 km over a horizontal distance of 100 km. What is the pressure gradient acceleration of an air parcel on the 500 hPa pressure surface, at a point midway between the two locations. (15 points)

(i) The key is to recognize that you can calculate the pressure gradient acceleration as due to a pressure difference on a geopotential height surface, or a geopotential height difference on a pressure surface. In this problem, you are given the geopotential height difference on a pressure surface, so the PGA $= -d\Phi/dy = -g_0 dZ/dy$, *where* $dy = 100$ *km, and* $dZ = 80$ *m.*

(ii) The slope of this pressure surface is oriented in the north-south direction. The latitude is 45 $^\circ$N. What is the direction and magnitude of the geostrophic wind? (Assume the local temperature is 270 K.) (15 points)

(ii) You set the PGA = CA (CA = Coriolis acceleration), or from the prognostic equation in the case $u = u_g$, $-fu_g - (1/\rho)dp/dy = 0$. *Now,* $(1/\rho)dp/dy = d\Phi/dy = g_0 dZ/dy$ *(equivalent ways of expressing pressure gradient acceleration). So* $-fu_g - gdZ/dy = 0$, *and solve for ug.*

(iii) If the surface pressure is 1000 hPa at both locations, estimate the mean temperature difference between the 500 hPa - 1000 hPa surfaces that gives rise to the 80 m change in geopotential height between the two locations. (15 points)

8. The diagram below shows the geopotential height at 1000 hPa Φ_{1000} (solid lines), the geopotential height at 500 hPa Φ_{500} (dashed lines), and the geopotential difference between the two pressure levels $\Delta\Phi_{1000-500}$ (dashed dot lines). At point A, draw the rough relative directions and magnitudes of the geostrophic winds at 1000 hPa and 500 hPa, and thermal wind of 1000-500 hPa layer. (8 points)

Geostrophic winds at 500 hPa and 1000 hPa should be roughly equal in magnitude, parallel to geopotential height contours, with larger geopotential height (warmer air) on the right. The thermal wind should go from the end of the 1000 hPa vector to the end of the 500 hPa vector.

9. (i) At 45° N on the 500 hPa surface, the geopotential decreases to the north at a rate of 2 J/kg every km. Calculate the geostrophic wind vector $\mathbf{V_g}$. (Assume $f = 1 \times 10^{-4}$ s^{-1}.)

Probably easiest to use:

$$u_g = -\frac{1}{f}\frac{\partial \phi}{\partial y}$$

$$v_g = \frac{1}{f}\frac{\partial \phi}{\partial x}$$

(iii) The actual wind vector at this location on the 500 hPa surface is $\mathbf{V} = (21, 1)$ m/s. Calculate the Coriolis acceleration vector. (12 points)

Use the expression below.

$$\frac{d\mathbf{V}}{dt} + f\mathbf{k} \times \mathbf{V} = -\nabla_{\mathbf{p}}\phi$$

$d\mathbf{V}/dt$ is equal to PGA + CA, so the CA is equal to $-f\mathbf{k} \times \mathbf{V}$.

(ii) Calculate the acceleration of the horizontal wind $d\mathbf{V}/dt$. (10 points)

You are given $\mathbf{V}$ and $\mathbf{V_g}$, so can find $\mathbf{V_{ag}}$. Then use:

$$\frac{d\mathbf{V}}{dt} = -f\mathbf{k} \times \mathbf{V_{ag}}$$

10. The figure above is taken from the text.

(i) Identify a location with a large positive ω, and indicate the direction of the geostrophic wind vector at this location. (4 points)

The geostrophic wind on a potential temperature surface is parallel to the Montgomery potential (solid lines). The largest ω is therefore when $\mathbf{V_g}$ is directed across isobars toward larger pressures (the dashed lines).

(ii) Identify a location with a large negative ω, and indicate the direction of the geostrophic wind vector at this location. (4 points)

(iii) Which location is most likely to be associated with clouds and/or precipitation? Explain. (4 points)

6. (i) At 45° N on the 500 hPa surface, the geopotential decreases to the north at a rate of 2 J/kg every km. Calculate the geostrophic wind vector $\mathbf{V_g}$. (Assume $f = 1 \times 10^{-4}$ s^{-1}.) (12 points)
(ii) The actual wind vector at this location on the 500 hPa surface is $\mathbf{V} = (21, 1)$ m/s. Calculate the acceleration of the horizontal wind $d\mathbf{V}/dt$. (14 points)
(iii) Would you expect the kinetic energy of the parcel to be increasing or decreasing? Explain. (4 points)

================ Thermal Wind =====================

3. The geostrophic wind at 600 hPa is 20 m/s directed to the north. The mean temperature of the 600 - 500 hPa layer decreases to the north at a rate of 1 K per 100 km. Let f = 1.0E-04.

(i) What is the thermal wind of the 600 - 500 hPa layer? Give (u, v) components. (12 points)

(ii) What is the average geostrophic wind vector $\mathbf{Vg} = (u_g, v_g)$ of the 600 - 500 hPa layer? (6 points)
(iii) What is the temperature gradient vector $\nabla\mathbf{T}$ of the 600 - 500 hPa layer? (8 points)
(iv) What is the rate of geostrophic temperature advection in the layer? Express in K/day. (10 points)

1. The mean temperature of the 750 - 500 hPa layer decreases northward by 3 C per 100 km. The 750 hPa geostrophic wind is from the south at 20 m/s. GIVE f.

(i) What are the (u,v) of the thermal wind of the 750 - 500 hPa layer? (4 points)

(ii) Estimate the (u,v) components of the geostrophic wind at 500 hPa. (6 points)

(iii) What is the geostrophic temperature advection in the 750-500 hPa layer? (4 points)

2. The figure below shows a cold core low. The solid lines show the 800 hPa geopotential height contours, and the dashed lines as contours are 1000 - 800 hPa layer isotherms. At the surface, the pressure decreases toward the center of the low. Would you expect the 800 hPa tangential winds to be stronger at 800 hPa than the surface, or weaker? Explain. (4 points)

3. Use a diagram to answer the following two questions. Explain why an appropriate force balance is, or is not, possible for these circulations. Show the direction and relative magnitudes of the pressure gradient, Coriolis, and centripetal accelerations. You can ignore friction.
(i) Is cyclonic (counter-clockwise) high possible in the Northern Hemisphere? (8 points)

No. Both PGA and CA point away from the high. No way to generate an inward centripetal circulation.

(ii) Is an anticyclonic (clockwise) low possible in the Northern Hemisphere? (8 points)

Yes. PGA and CA both point inward, so inward centripetal acceleration can be generated from sum of two accelerations. This corresponds to saying that it ispossible to stir a glass of tea in the clockwise direction.

4. The figure below shows geopotential height on the 900 hPa surface. The low pressure system on the left is cold core, while the low pressure system on the right is warm core. In a cold core system, air at the center is cold relative to air on the same pressure level, further from the low. In a warm core system, air at the center is warm relative to air on the same pressure level, further from the low. Air parcels are undergoing circular rotation about the low. Assume that the pressure gradients are the same for each system and ignore friction.
(i) On the diagrams at point A, indicate the rough direction of the gradient wind at 900 hPa for each system. (2 points)

The gradient wind will point cyclonically (counterclockwise) in each case.

(ii) On the diagrams at point A, indicate the rough direction of the thermal wind in the lower troposphere for each system. (4 points)

The thermal wind has warm on the right, so is cyclonic for cold core system and anticyclonic for warm core system.

(iii) In which system does the wind speed typically increase with height? Explain. (4 points)

cold core system: thermal wind adds to gradient (geostrophic) wind at pressure level closer to the surface.

(iv) What is an example of a warm core system that has a significant effect on weather? (3 points)

hurricanes

(v) What is an example of a cold core system that has a significant effect on weather? (3 points)

mid-latitude cyclones

=================== **Hurricanes** ====================

1. The tangential speed about a hurricane is given by $V(r) = ar^2$, where a is a constant, and r the distance from the center of the hurricane. You can assume that the frictional force is zero. Air parcels in the hurricane are undergoing circular motion.

(i) Draw a diagram of a parcel in the hurricane showing the three way force balance between the pressure gradient, Coriolis, and centripetal accelerations. (2 points)

(ii) What is the the radial variation in angular velocity $\omega(r)$? (2 points)

(iii) The surface pressure at the center of the hurricane is p_0. Derive an expression for the radial variation of surface pressure $p(r)$ in terms of p_0, a, f, and ρ_0 (the density of the atmosphere, assumed constant) (6 points)

2. Two hurricanes have the same radial distribution of geopotential height about their center $\phi(r)$, but are at different latitudes. Which hurricane would you expect to have the stronger horizontal winds: the hurricane closer to the equator or the hurricane further north? You can assume that both hurricanes are in the Northern Hemisphere, that friction can be ignored, and that the horizontal winds are circular and obey gradient wind balance. Explain your choice with a diagram. (12 points)

The PGA would be the same for both. The inward centripetal acceleration is equal to PGA - CA (CA points out). Further north, where the CA is larger, the centripetal acceleration will be smaller. This implies a smaller V.

In an ordinary low, under gradient wind balance, the coriolis and centripetal accelerations balance the pressure gradient acceleration. If $\phi(r)$ is the same for the two hurricanes, the pressure gradient acceleration is the same. For a fixed PGA = CF + CE, if CF is larger further from the equator, then gradient wind balance is achieved with a smaller CE, i.e. the wind speed must be smaller. It is much simpler to explain this way than using a formula (hence asked for a diagram).

3. The central surface low pressure of a hurricane is often comparable with those of mid-latitude lows. However, wind speeds near the surface within a hurricane are typically stronger than a mid-latitude low pressure system. What is the explanation? (2 points)

4. Would you expect the wind speeds inside a hurricane to be close to the geostrophic wind? Why or why not? (5 points)

No, wind speeds in a hurricane would not usually be close to the geostrophic wind. One could say the Rossby number of a hurricane would not be close to zero; they occur in the tropics where the geostrophic approximation is generally poor; that the vertical velocities inside the deep convective towers would be much larger than the 1 cm/sec assumed in the scaling analysis justifying the geostrophic approximation; they have faster horizontal wind speeds than the 10 m/sec assumed in scaling; smaller horizontal dimensions that the assumed spatial scale of 1000 km, and finally, that the strongest wind speeds in a hurricane occur close to the boundary layer, where they are more likely to be affected by *friction.*

5. The boundary layer of a hurricane extends from the surface to 2 km. The vertical velocity at the surface is zero, while $w = 1$ m/s at the top of the boundary layer. The vertical velocity increases linearly with height from the surface to the top of the boundary layer. The hurricane is at a latitude of 10 $^{\mathrm{o}}$, where $f = 2.53 \times 10^{-5}$ s-1.

(i) What is the value of the divergence in the boundary layer? (8 points)

(ii) Suppose a parcel enters the hurricane boundary layer with $\xi = 0$ originally, and resides in the boundary layer for one hour. What is the value of the relative vorticity ξ of the air parcel at the end of 1 hour? (20 points)

========== **Old Questions: No Longer Directly Relevant** ===========

1. Draw a diagram of the Ekman spiral solution for a boundary layer in which $(u, v) = (0, 0)$ at the surface and (u, v) asymptotically approaches $(u_g, 0)$ above the boundary layer. Show the spiral solution in the u/u_g and v/v_g plane, where u/u_g is the horizontal axis, and v/v_g is the vertical axis. Show the location along the curve where $z = 0$, and where $z = D_e$. (4 points)

2. Do air parcels often execute free momentum oscillations in the atmosphere? Explain. (6 points)

3. (i) What is the main assumption of the Boussinesq equations? (2 points)

(ii) Why is it reasonable under most circumstances? (2 points)

3. The water in a spinning vessel is rotating at constant rate of 10 revolutions per minute. The flow is laminar except for a narrow Ekman Layer near the bottom. The mean depth of the fluid is 0.30 m. The eddy viscosity coefficient is $K_m = 1 \times 10^{-6}$ m^2/s.

(i) What is the angular velocity (call it Ω) of the water in the vessel? (2 points)

(ii) What is the depth of the Ekman layer D_e? (Hint: in a rotating vessel the Coriolis parameter f can be taken to be equal to the angular velocity.) (4 points)

(iii) What is the (relative) vorticity of the spinning water? (2 points)

(iv) What is the approximate timescale over which the vorticity in the vessel decreases by 1/e via the frictionally induced secondary circulation? (4 points)

(v) Explain why is it reasonable to make the association between f and Ω in this problem? (2 points)

(vi) Within the Ekman Layer, how would you expect the fluctuations in vertical velocity (w') to be correlated with fluctuations in tangential velocity (u'). Explain. (2 points)