PHYS/OCEA 4411/5411 A Atmospheric Dynamics II Old Test Questions

================ **General** =================

1. What are two important approximations made in the derivation of the quasi-geostrophic equations?

2. In what season of the year would you expect forecasting over the continental United States to have the lowest predictability? Briefly discuss a reason for why this decreased predictability may occur.

Predictability is reduced in the summer due to deep convection.

3. Specify three conditions in which you would expect the actual wind to more closely approximate the geostrophic wind.

4. Both hurricanes and mid-latitude low pressure systems are characterized by convergent inflow of mass near the surface. The low level inflow of mass would ordinarily increase surface pressure, unless compensated by some other process. This question addresses the issue of how developing and mature hurricanes avoid an increase in surface pressure. (Hint: hurricanes rain a lot.)

(i) Would you expect the geopotential height anomalies above a hurricane (say on the 200 hPa surface) to be positive or negative? Explain your choice using a relevant equation from the formula sheet, or another equation of your choice.

(iii) How might these upper tropospheric geopotential height anomalies help reduce surface pressure?

(iv) Would you expect the circulation changes associated with these upper tropospheric height anomalies to increase or decrease the relative vorticity above a hurricane? Explain your choice with respect to a particular equation.

5. In the case of geostrophic warm advection, would you expect

$$\mathbf{V}_g \cdot \nabla \left(\frac{\partial \Phi}{\partial p} \right)$$

to be positive or negative? Explain in detail. A diagram may help. Be careful with signs. Show your argument as a sequence of logical point form steps if possible.

Positive. Geostrophic warm advection implies $\mathbf{V}_g \cdot \nabla T < 0$*. Then substitute:*

$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p}$$

6. (i) What is an important approximation associated with the following horizontal momentum equation on a pressure surface?

$$d\mathbf{V}/dt + f\mathbf{k} \times \mathbf{V} = -\nabla_{\mathbf{p}}\phi$$

The best answer is probably neglect of friction.

(ii) Use the momentum equation from (i) to derive the following expression for the ageostrophic wind. Show and justify all steps.

$$\mathbf{V}_{ag} = \frac{\mathbf{k}}{f} \times \frac{d\mathbf{V}}{dt}$$

In notes.

1. The geopotential on the 250 hPa pressure surface has the following variation, with x the east-west coordinate and y the north-south coordinate. Assume that $f = f_0$ and can be treated as a constant.

$$\Phi(x, y) = \cos(x) - y$$

(i) Suppose $(x, y) = (\pi, 0)$. Does this location correspond to a ridge, a trough, half way between a ridge and the next trough, half way between a trough and the next ridge, or none of these? Explain. (1 point)
(ii) Calculate the geostrophic wind vector $\mathbf{V_g} = (u_g, v_g)$. (2 points)
(iii) Determine an expression for the geostrophic relative vorticity ξ_g in terms of f and x. (2 points)
(iv) Determine an expression for the gradient vector $\nabla(\xi_g + f)$. (2 points)
(v) Assume that the ξ_g pattern is stationary, i.e.,

$$\partial\xi_g/\partial t = 0.$$

Obtain an expression for the horizontal divergence $\nabla \cdot \mathbf{V}$ from the QG Vorticity equation. (Note that this will not in general equal the divergence of the geostrophic wind.) (2 points)
(vi) Suppose $(x, y) = (\pi/2, 0)$. Does this location correspond to a latitude where the divergence is maximized, minimized, equal to zero, or none of these possibilities? (1 point)
(vi) With respect to the geopotential height pattern given above, does this same location $(x, y) = (\pi/2, 0)$ correspond to a ridge, a trough, half way between a ridge and the next trough, half way between a trough and the next ridge, or none of these? Explain. (1 point)

================ Vertical Motion =================

1. There is convergence in the upper troposphere, and divergence in the lower troposphere. Show how you would expect the vertical velocity ω to vary with pressure in this situation. Pressure should be along the vertical axis with $p = 1000$ hPa denoting the surface, and $p = 200$ hPa denoting the tropopause.

2. We have often utilized the following assumption for mid-latitude synoptic systems. Explain or justify this statement. Note: $\propto$ means "proportional to". Showing a typical vertical profile of ω for a developing low pressure system would help your argument.

$$\frac{\partial^2}{\partial p^2}\omega \propto -\omega$$

Draw the profile of ω from the notes. Note that the vertical gradient of ω near the surface is negative (i.e $-\frac{\partial \omega}{\partial p} < 0$.) However, vertical gradient in upper troposphere is positive, so $\frac{\partial^2 \omega}{\partial p^2} > 0$. Then note that $\omega < 0$, and have your result.

1. Assume $\omega = 0$ on the 200 hPa surface, and the following divergence profile:

200 - 0 hPa: $\nabla \cdot \mathbf{V} = 0$

300 - 200 hPa: $\nabla \cdot \mathbf{V} = 2$/day

800 - 300 hPa: $\nabla \cdot \mathbf{V} = 0$

1000 - 800 hPa: $\nabla \cdot \mathbf{V} =$ -0.5/day

(i) Calculate ω at 300 hPa, 800 hPa, and 1000 hPa (in units of hPa/day).
(ii) Suppose $p_s = 1000$ hPa. Estimate the local tendency in surface pressure due to this divergence profile. (2 points)

================ **Vorticity** =================

1. The figure below shows a surface low downstream of an upper level trough (similar to Figure 6.7 in the text).

(i) Would you expect $\mathbf{V}_g \cdot \nabla(\xi_g + f)$ to be positive or negative above the surface low? Explain.

The tip of the trough is typically a vorticity maximum, so you would expect positive vorticity advection downstream of the trough over the low. This term should be negative.

2. Derive: $\xi_g = \frac{1}{f_0}\nabla^2\Phi$
In the notes

3. The absolute vorticity following the motion of a parcel is usually more nearly conserved in the mid-troposphere than the upper or lower troposphere. Explain.

4. Specify the three sources of η following the motion of an air parcel.

5. Make the following assumptions:
(i) The relative vorticity pattern in the atmosphere is stationary (i.e. $\partial\xi/\partial t = 0$).
(ii) You can ignore the solenoidal, tilting, and vertical advection terms in the vorticity tendency equation.
(iii) Horizontal advection is decreasing ξ in the upper troposphere and increasing ξ in the lower troposphere.
(iv) The vertical velocity at the surface and tropopause is zero.
Given these assumptions, would you expect mean upward or downward motion at mid-levels? Explain in detail with reference to relevant equations. A diagram may help. It is preferable to explain without using QG Equations, which are approximations.

The dominant terms in the vorticity budget must be horizontal advection and divergence. In the upper troposphere, if horizontal advection is is decreasing ξ, and ξ is constant, then the divergence term must be increasing ξ, i.e. need a convergence. In the lower troposphere, need a divergence to decrease ξ and offset horizontal advection. A divergence in the lower troposphere and a convergence in the upper troposphere imply downward motion at mid-levels. The main problem here was a reading problem. People interpreted (iii) as implying that ξ was decreasing ξ in the upper troposphere, rather than a s a reference to a particular tendency. I tried to mark a bit generously. I did take points off for people who used QG theory like the omega equation. This question is typical of QG thinking, but it is better to not use QG and just explain with reference to the more fundamental vorticity equation.

======= **Chapter 6: Ageostrophic Flow at Ridge-Trough-Ridge** ============

1. Assume that the speed of the geostrophic wind is constant through the ridge - trough - ridge sequence in the upper troposphere shown below. Note:

Point A is halfway between the first ridge and the trough.

Point B is halfway between the trough and the second ridge.

Draw the acceleration vector $d\mathbf{V_g}/dt$ at the following four locations:

(i) slightly downstream of the first ridge, (1 point)

(ii) slightly upstream of the trough, (1 point)

(iii) slightly downstream of the trough, (1 point)

(iv) slightly upstream if the second ridge. (1 point)

(v) Show the direction of the ageostrophic wind at the four locations above.

(vi) Would expect upper level divergence or convergence at points A and B? Explain.

In the notes/text.

=== Chapter 6: Ageostrophic Flow/Thermal Wind Balance at Jet Entry/Exit ===

1. The figure below shows a jet in the Northern Hemisphere upper troposphere. The solid lines are lines of constant wind speed (isotachs). The dashed lines are lines of constant geopotential.

(i) Show the direction of $d\mathbf{V}/dt$ at points A and A′.

(ii) Show the expected direction of $\mathbf{V_{ag}}$ at points A and A′.

(iii) Show the expected direction and relative magnitude of of $\mathbf{V_{ag}}$ at points B and B′

(iv) Show the expected direction and relative magnitude of of $\mathbf{V_{ag}}$ at points C and C′.

(v) Show on the figure whether you would convergence (CON) or divergence (DIV) at the following four locations: between A and B, between A and C, between A′ and B′, and between A′ and C′

(vi) The figure below shows a vertical cross section of the jet exit region along the line between C′ and B′. Use vertical arrows to show whether you would expect an upward or downward secondary ageostrophic circulations at points X and Y.

(vii) Are the vertical secondary ageostrophic circulations you have shown in the previous question part of a thermally direct or indirect circulation? Explain.

(vii) Indicate the expected directions of the secondary ageostrophic horizontal circulations at the jet axis (J), and the point Z.

The next several questions address the role of the primary (geostrophic) and secondary (ageostrophic) circulations in creating and/or destroying thermal wind balance at the jet exit, with reference to the following equation.

$$p\frac{\partial u_g}{\partial p} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p$$

(viii) Would you ordinarily expect $\partial u_g/\partial p$ to be positive or negative in the mid-troposphere below the jet? Explain.

(ix) Would you ordinarily expect $\partial T/\partial y$ to be positive or negative in the mid-troposphere below the jet? Explain.

(x) Consider the left hand side of the above equation. In the jet exit region, would you expect geostrophic advection at the jet level to increase this term (make it more positive) or decrease this term (make it more negative)? Explain.

(xi) Consider the right hand side of the above equation. In the jet exit region, would you expect the geostrophic advection to increase this term (make it more positive) or decrease this term (make more negative)? Explain.

(xii) Given your answers to the above questions, does geostrophic advection tend to destroy or maintain thermal wind balance? Explain.

2. (i) The figure below labeled "Jet Exit" shows lines of constant geopotential in the upper troposphere (solid) and thickness contours between the upper and lower troposphere (dashed) of a typical jet exit region, with the jet oriented from west to east. Explain in words why this particular geostrophic wind configuration would tend to destroy thermal wind balance at point B, if thermal wind balance is obeyed initially. That is, explain why the geostrophic wind is destroying:

$$f_0 \frac{\partial u_g}{\partial p} = -\frac{\partial^2 \phi}{\partial y \partial p}$$

Discuss the forcings on both momentum and temperature, and explain how they change the RHS and LHS of the above equation.

The signs here were more complicated than I thought. I think this is the reason the text uses a jet oriented in the north-south direction in its explanation of how the geostrophic flow undermines TWB at the jet exit and entrance. I wouldn't expect all of this discussion, but something along these lines. The LHS is usually negative in the troposphere, since u_g decreases toward the surface (the positive p direction). Advection of higher u_g air from the jet center will increase u_g and make the LHS even more negative. The RHS is even more complicated. It helps to write:

$$-\frac{\partial^2 \phi}{\partial y \partial p} = -\frac{\partial}{\partial y}\frac{\partial \phi}{\partial p}$$

Since the geopotential increases with height, $\partial\phi/\partial p$ is negative. Φ decreases in the positive y direction (in the NH usually). The negative sign out front means we have three negatives, so the RHS is usually negative also. Diffluence at the jet exit will weaken the Φ gradient and decrease the rate of decrease of Φ with y. Therefore diffluence will tend to make the RHS less negative, i.e. increase it. Since the LHS and RHS are being modified in opposite ways, TWB as expressed in this equation is being undermined.

One way to avoid the sign confusion is to talk in terms of magnitudes. Momentum advection increases the magnitude of the LHS, but decreases the magnitude of the RHS.

(ii) Use the following expression to indicate the approximate amplitude and direction of the ageostrophic wind at points A, B, and C.

$$\mathbf{V}_{ag} = \frac{\mathbf{k}}{f} \times \frac{d\mathbf{V}}{dt}$$

(iii) Indicate the approximate locations of convergence and divergence along the A - C line at the jet level.

(iv) Explain how the mid-tropospheric ageostrophic vertical motion in a jet exit region helps maintain thermal wind balance.

3. The figure below shows a linear jet in the upper troposphere. The contours represent isotachs of constant wind speed. Indicate the likely directions of the ageostrophic wind at locations A and B.

4. The figure below shows a jet entry region. The dashed contours are lines of constant 1000 - 500 hPa geopotential height thickness Φ'. The solid contours are lines of constant 500 hPa geopotential height.

(i) Express the condition for thermal wind condition balance at point A.

This question is based on Figure 6.8. I was looking for the equation immediately following (6.35a) in the text. However the question was not phrased specifically enough, so accepted more general answers.

(ii) This wind configuration is said to destroy geostrophic balance. Explain using mathematical reasoning.

Confluence tends to push thickness line closer together, increasing the thermal wind. However geostrophic momentum advection will lower the actual wind at point A. The pushing apart of the actual and thermal wind, undermines geostrophic balance. Best to express with reference to equation in text following (6.35a).

4. The figure below shows a northward jet in the Northern Hemisphere. (This figure is from the notes. The 500 hPa geopotential height is confluent and the 1000 - 500 hPa thickness lines are straight north south with the warm air on the right. Point A is near the bottom along the axis to the jet, while point C is closer to the jet maximum.

(i) Indicate the direction and relative strengths of the pressure gradient acceleration and geostrophic wind vectors at points A and C. (8 points)

(ii) With respect to the equation $fv_g = \partial\phi/\partial x$, discuss how gesotrophic advection of the gesotrophic wind, and temperature, would be expected to undermine geostrophic balance at location C over time. Make sure sure to discuss the impact of advection on both sides of the equation. (8 points)

====== **Chapter 6: Sutcliffe Low: Propagation ALONG THERMAL WIND** ====

1. The figure below shows a low near the surface in a baroclinic environment. Dashed lines indicate 1000 - 500 hPa geopotential height thickness, while the solid curves represent surface isobars. Let $\mathbf{V}_0 = (u_0, v_0)$ refer to the surface wind (assumed geostrophic), and let $\mathbf{V}_s = (u_s, v_s)$ refer to the thermal wind vector of the 1000 - 500 hPa layer, which can be assumed constant in time. The x and y directions are as indicated.

(i) For this geometry, for a vertical line through the center of the low, derive the following equation. Make sure to show all steps, and to explain all assumptions.

$$\left(\mathbf{V}_s \cdot \nabla\right)\mathbf{V}_0 = v_s \frac{\partial u_0}{\partial y}\mathbf{i}$$

In notes.

(ii) On the diagram, indicate the direction of the $\left(\mathbf{V}_s \cdot \nabla\right)\mathbf{V}_0$ vector at the center of the low.

In notes.

(iii) On the diagram, indicate the expected direction and approximate relative magnitude of the 1000 - 500 hPa layer ageostrophic vector $\mathbf{V}_{ag,U} - \mathbf{V}_{ag,L}$ at the center of the low, at a point directly north of the center of the low, and at a point directly south of the center of the low.

In notes.

(iv) Along a vertical line through the surface low, indicate the location on the figure where you would expect 1000 - 500 hPa layer convergence, and where you would expect 1000 - 500 hPa layer divergence?

In notes.

(v) In what direction therefore would you expect this surface low to propagate? Explain.

The low will tend to propagate in the direction of layer divergence, since this will tend to export mass from the column and reduce surface pressure.

(vi) The actual rule of thumb weather forecasters use to predict the direction of propagation of surface lows is somewhat modified from the prediction associated here with Sutcliffe. What is this modification, and why might it be required?

The answer I was looking is that lows tend to propagate 30 degrees to the left of the shear vector, due to rotation of the background baroclinicity associated with warm advection and cold advection of the low. Accepted other answers if sounded reasonable.

============= **Chapter 6: Sutcliffe and Frontogenesis** ===========

1. In the figure below, dashed contours are lines of constant 1000 - 500 hPa geopotential height thickness Φ'. The solid contours are lines of constant 500 hPa geopotential height. The confluent geostrophic wind is in the process of tightening the east-west thermal gradient in the layer.

(i) Show the direction of the local 1000 - 500 hPa thermal wind vector V_s at point A.

(ii) Show the direction of the dV_s/dt vector at A.

(iii) Indicate the direction of the 1000 - 500 hPa layer ageostrophic wind vector at A. Explain your choice using the following Sutcliffe expression for the layer ageostrophic wind, assuming that $\left(\mathbf{V}_s \cdot \nabla\right)\mathbf{V}_0 = 0$.

$$\mathbf{V}_{ag,U} - \mathbf{V}_{ag,L} = \frac{\mathbf{k}}{f} \times \left[\left(\mathbf{V}_s \cdot \nabla\right)\mathbf{V}_0 + \frac{d\mathbf{V_s}}{dt}\right]$$

(iv) Assume that $d\mathbf{V_s}/dt$ goes to zero in the east and west directions away from point A. Show how you would expect the layer ageostrophic wind to vary away from A in the east and west directions, and indicate the expected locations of 1000 - 500 hPa layer convergence and divergence.

(v) In what sense can the induced ageostrophic vertical circulation in this wind configuration be said to be trying to maintain thermal wind balance? Explain with respect to the following thermal wind equation.

$$f_0 \frac{\partial v_g}{\partial p} = \frac{\partial^2 \phi}{\partial x \partial p}$$

2. In the figure below, dashed contours are lines of constant 1000 - 500 hPa geopotential height thickness Φ'. The solid contours are lines of constant 500 hPa geopotential height.

(i) How would you expect the 1000 - 500 hPa thermal wind vector at A to respond to this flow configuration? Explain using mathematical reasoning.

I was looking for some elements of the discussion around Figure 6.4 in the text. V_s accelerates in the same direction as V_s due to increase in thickness gradient due to confluence.

(ii) Indicate the direction of the 1000 - 500 hPa ageostrophic wind vector at A due to a tendency in the 1000 - 500 hPa thermal wind. Explain your choice using mathematical reasoning.

This question refers to a layer thermal wind. Therefore the best answers involved the Sutcliffe approach, especially Equations like (6.6) and how this difference in acceleration between the top and bottom of a layer is related to the layer ageostrophic wind.

(iii) Indicate the locations of 1000 - 500 hPa layer convergence and divergence. Explain using mathematical reasoning.

Comes from spatial pattern of layer ageostrophic wind to west and east of point A.

(iv) In what sense can the induced ageostrophic circulation in this wind configuration be said to trying to maintain thermal wind balance? Explain using mathematical reasoning.

Upward motion to east of A + downward motion to west of A $\rightarrow$ decrease in thickness gradient $\rightarrow$ decrease in thermal wind $\rightarrow$ smaller discrepancy between actual 1000 - 500 hPa layer wind and thermal wind $\rightarrow$ smaller violation of geostrophic thermal wind balance.

============= **Chapter 6: Isallobaric Wind** =================

8. Starting from :

$$\mathbf{V}_{ag} = \frac{\mathbf{k}}{f} \times \frac{d\mathbf{V}}{dt}$$

Prove:

$$\mathbf{V}_{ag,I} = -\frac{1}{f^2}\nabla\frac{\partial\,\Phi}{\partial t}$$

Remember that the isallobaric wind $\mathbf{V}_{ag,I}$ is one of three components of a particular decomposition of the ageostrophic wind. You are permitted to make a substitution of $\mathbf{V_g}$ for $\mathbf{V}$.

In the notes and text.

======== **Chapter 6: ω Equation for Diagnosing Vertical Velocity** =====

1. (i) Let ξ_g refer to the geostrophic vorticity on an upper troposphere pressure surface. Let ξ_{g0} refer to the geostrophic vorticity on a pressure surface in the lower troposphere. Let Φ' refer to the difference in geopotential between the upper and lower tropospheric pressure surfaces. Suppose that at some fixed horizontal location, the difference $\xi_g - \xi_{g0}$ is increasing with time. Prove that

$$\nabla^2\frac{\partial\Phi'}{\partial t} > 0$$

In notes.

(ii) Suppose that $\partial\Phi'/\partial t$ is a local extremum. If the result in (i) is true, which is more likely: a local maximum or local minimum? Explain.

In notes.

(iii) Use your answer to (ii) to predict which is more likely: mean upward or downward motion between the upper and lower troposphere? Explain.

Upward motion to decrease geopotential thickness.

2. Consider the following version of the ω equation.

$$\sigma\left(\nabla^2 + \frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\,\nabla\cdot\mathbf{Q}$$

(i) Suppose that ω is zero at the surface and at the tropopause, and reaches a maximum negative value in the mid-troposphere. Would you expect the left hand side of the above equation to be positive or negative? Explain. Ignore horizontal variations in ω. A diagram may help.

The sign of $\partial^2\omega/\partial p^2$ is positive. Starting from the tropopause (low p), and going toward the surface (high p), $\partial\omega/\partial p$ is initially negative, but the slope increases toward the surface, so the second derivative is positive. Another valid answer is that the second derivative of a sinusoidal function changes the sign.

(ii) Assume that the vertical motion is upward in the mid-troposphere. Would you typically expect $\nabla \cdot \mathbf{Q}$ to be positive or negative? Explain.

In (i) we had upward motion in the middle troposphere with LHS > 0. In this case, LHS > 0 $\Rightarrow -\nabla \cdot \mathbf{Q} > 0 \Rightarrow \nabla \cdot \mathbf{Q} < 0$.

3. What is one weakness or limitation of the omega equation in describing cyclogenesis?

Variety of answers: diagnostic rather than prognostic, no diabatic heating, assumptions of QG theory.

4. The advection of geostrophic vorticity ξ_g by the geostrophic wind $\mathbf{V_g}$ is stronger at some upper level than near the surface.

(i) How would you expect the geopotential thickness Φ' between the upper and lower levels to respond to this vertical shear in vorticity advection. Explain in as mathematical manner as possible.

I was looking for a summary of the argument given on the bottom half of page 162 in the text, and in a bit more detail, in the notes. As discussed in class, I am not sure that the conclusion $\frac{\partial}{\partial t}\Phi < 0$ *can be justified. Better to say that the local time tendency of the geopotential height thickness is a local minimum.*

(ii) In the context of the ω equation, how would you expect the vertical motion of the layer to respond to this shear in vorticity advection? Explain in as mathematical manner as possible.

Greater PVA aloft $\rightarrow$ *relative reduction in* Φ' $\rightarrow$ *accomplished (at least in part) via induced upward motion (Obviously want to make specific reference to* ω *equation in this argument).*

(iii) Would you expect the induced vertical motion of the layer to enhance or diminish the difference in the local tendencies of the upper and lower geostrophic vorticities associated with geostrophic advection? Explain in as mathematical manner as possible.

induced upward motion $\rightarrow$ *upper level divergence + lower level convergence* $\rightarrow$ *upper level vorticity destruction + lower level vorticity production* $\rightarrow$ *reduction in vertical vorticity shear increased by horizontal advection (Should refer to relevant equations here, especially QG vorticity equation).*

==== Chapter 6: Destruction of Thermal Wind Balance by Geostrophic Flow =====

1. (i) Draw the lines of constant geopotential, and thickness contours, of a typical jet entry region, with the jet oriented from west to east. The thickness contours should be indicated using dashed lines with the warm air to the south and the cold air to the north of the jet core. Explain in words why this particular wind configuration tends to destroy thermal wind balance. That is, explain why the wind is destroying either:

$$f_0 \frac{\partial v_g}{\partial p} = \frac{\partial^2 \phi}{\partial x \partial p}$$

or:

$$f_0 \frac{\partial u_g}{\partial p} = -\frac{\partial^2 \phi}{\partial y \partial p}$$

Please discuss the forcings on both momentum and temperature.

Looking for discussion around Fig 6.8.

(ii) Show the induced ageostrophic circulation of this wind configuration (both vertical and horizontal). Discuss how this induced circulation helps maintain thermal wind balance.

Wanted to see some discussion of how northward v_{ag} at jet axis entry region, times f, is a momentum source to the jet core. By this, I mean that the Coriolis acceleration fv is a term in du/dt, and in this case along the jet axis $v = v_{ag}$, so the eastward acceleration in the jet entry comes from the northward ageostrophic wind. Most of you correctly discussed how the induced direct frontolytic circulation would offset the horizontally frontogenetic circulation (confluence at jet entry), and help restore thermal wind balance.

======== Chapter 6: Diagnosing Vertical Motion from Q Vectors =========

1. The geopotential distribution is given by $\Phi(x, y) = xy$. The temperature distribution is given by $T(x, y) = x^2$.

(i) Solve for the geostrophic wind vector $\mathbf{V_g}$.

(ii) Solve for the temperature gradient vector $\nabla\mathbf{T}$.

(iii) Solve for the $\mathbf{Q}$ vector $\mathbf{Q} = (Q_1, Q_2)$. You can treat f as a constant.

(iv) Solve for the divergence of the Q vector, $\nabla \cdot \mathbf{Q}$.

(v) Would you expect upward, downward, or zero vertical velocity, based on your solution for the Q vector? Explain with reference to an equation.

============ Chapter 6: Obtaining Q Vectors ===================

1. The figure below shows a jet entry region. Dashed lines represent isotherms, while the solid lines indicates contours of constant geopotential height.

(i) Indicate the direction of the $\mathbf{Q}$ vector at point A. Give a detailed explanation of your choice using the expression for the $\mathbf{Q}$ vector in terms of natural coordinates. Draw in all relevant intermediary variables on the diagram, and indicate the directions of the local coordinate vectors.

(ii) Based on the spatial variation of the $\mathbf{Q}$ vector, would you expect upward or downward motion at points B and C? Explain with reference to a relevant equation.

(iii) One of the conditions for thermal wind balance at point A is:

$$p\frac{\partial u_g}{\partial p} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p$$

The geostrophic circulation shown in this figure is said to destroy thermal wind balance. Specify the two ways (one for each side of the above equation) in which the horizontal geostrophic flow undermines the validity of thermal wind balance at point A.

(iv) The induced transverse and vertical ageostrophic circulation in jet entry region is said to help maintain thermal wind balance. It does this in two ways. Briefly discuss each of these two ways.

=============== Chapter 7: Jets and Frontogenesis ============

1. You are in an exit region of a westerly jet. The jet is undergoing upper tropospheric frontogenesis.

(i) Under the jet maximum, would you expect $\partial u/\partial z$ to be positive or negative? Explain.

The answer is perhaps simpler than most people treated the question. I guess part of the problem may have been that by "under" here I mean below, not to the south (I am asking for a shear in the vertical direction). Under a westerly jet, will always have $\partial u/\partial z > 0$.

(ii) In what direction would the vorticity associated with this wind shear point? Explain with reference to the right hand rule (or other way of determining vorticity direction).

Points north. In the class notes.

(iii) Suppose there is a thermally indirect circulation in the jet exit region. Would you expect the vertical motions associated with this circulation to increase ξ or decrease ξ? Explain with reference to the vorticity discussed in (ii), a diagram showing a vertical cross-section of the jet, and the expected vertical motions in the jet exit region.

Thermally indirect means sinking on the equatorward side of the jet, and rising motion on the poleward side. This would tilt the northward pointing vorticity from (ii) into the vertical and increase ξ. *Some of you discussed the convergence and divergence associated with the vertical motions, and how this would affect* ξ, *but the question was asking for a vorticity change coming from the vorticity source discussed in (ii).*

2. These questions are related to a discussion in class on the how frontogenesis and jet dynamics are connected.

Comment: The point of this question is that if you have some process that increases the meridional baroclinicity under a westerly jet, you will have a strengthening of the jet by thermal wind balance. This also implies an increase of relative vorticity on the poleward side, and a decrease on the equatorward side. These vorticity changes have to come from somewhere. In some cases, the required vorticity changes are provided by convergence (to increase vorticity) or divergence (to decrease vorticity). Applying this logic here results in a thermally direct circulation. This response is more characteristic of jet entry than jet exit regions (various arguments determining the ageostrophic wind). In jet exit regions, the required vorticity source could be tilting by the thermally indirect circulation of the northward pointing vorticity under the jet.

(i) Suppose the usual south to north baroclinicity (temperature gradient) beneath a westerly upper tropospheric jet were to increase. Would you expect the speed of the jet to increase or decrease? Explain (mathematically if possible).

Thermal wind would increase, so westerly jet would increase.

(ii) Would expect the relative vorticity on the poleward side of the jet to increase or decrease. Explain (mathematically if possible).

$\partial u/\partial y$ *would decrease on the poleward side, so vorticity would increase, using the definition of relative vorticity.*

(iii) Would you expect divergence or convergence on the poleward side of the jet? Explain (mathematically if possible).

Would expect convergence to supply, or "spin up", (at least in part) the vorticity increase.

(i) Would you expect upward or downward motion under the poleward side of the jet? Explain (mathematically if possible).

Convergence plus the tropopause lid will drive downward motion.

(v) Would you expect the potential vorticity under the poleward side of the jet to increase or decrease?

Descent will bring down high PV air from the lower stratosphere.

1. (i) Suppose the meridional temperature gradient under a westerly (eastward) jet in the upper troposphere were to increase. How would you expect the geostrophic zonal wind speed of the jet to respond to this temperature change? Explain with respect to a *specific equation*, and give the signs of the relevant variables.
(ii) In response to this temperature gradient increase, would you expect the relative vorticity on the poleward side of the jet to increase or decrease? Explain with respect to a specific equation, or definition, and indicate the changes in the relevant terms.

(iii) Assume that the local rate of change in relative vorticity at the jet level is given by the quasi geostrophic vorticity equation, and that the horizontal advection of vorticity is zero. In this case, how is the vorticity change on the poleward side of the jet generated?
(iv) Given your answer to (iii), would you expect upward or downward motion under the jet on the poleward side? Again, explain with respect to a *specific equation*, and discuss the changes in the relevant terms, and employ an appropriate boundary condition we have usually assumed valid at the tropopause.

============= Chapter 7: Frontogenesis ===================

1. (i) Draw streamlines of the two-dimensional velocity field $\mathbf{V} = (y, x)$ in the (x, y) plane. Indicate the axis of contraction and the axis of dilation.

(ii) Calculate the parameters D, F_1, and F_2 of this horizontal wind field, as defined in the expression for $\Im_{2D}$.

(iii) Suppose that this velocity field is acting on the temperature field $\theta(x, y) = xy$. Calculate the gradient vector $\nabla\theta$.

(iv) Calculate $\Im_{2D}$ using your answers for D, F_1, F_2, and $\nabla\theta$.

2. (i) Draw streamlines of the two-dimensional velocity field $\mathbf{V} = (y, x)$ in the (x, y) plane. Indicate the axis of contraction and the axis of dilation.

The axis of dilation is along the $y = x$ direction.

(ii) What is the horizontal divergence of this velocity field?

Easy to show it is zero.

(iii) Suppose that this velocity field is acting on the temperature field $\theta(x, y) = y$. Would you expect $\Im_{2D}$ to be positive, negative, or zero. Explain.

The simplest answer is to note that $\Im_{2D} = 0$ since the divergence is zero and β is 45 degrees. You could also note that $F_1 = 0$. F_2 is not zero but it multiplies a term that is zero in the expression for $\Im_{2D}$.

======== Chapter 7: Geostrophic Flow along a Front ============

======== Fluid with Sloping Density Discontinuity ===============

1. The figure below shows a layer of fluid with constant depth H. The fluid is divided into a warm region (density ρ_w) and a cold region (density ρ_c). The boundary between the two fluids is a density discontinuity and has a slope as drawn. The pressure at the top of the fluid (i.e. at $z = H$) is zero. The fluid is gravitationally stable and the pressure at each point in the fluid is equal to the hydrostatic pressure. Assume that the front extends for a distance L in the horizontal (e.g. so that the slope of the front equals H/L). The y axis points to the right, while the positive x direction is out of the page. Region I refers to the fluid in the warm water. Region II refers to the cold water directly below the front. Region III refers to the cold water to the right of the front. Let $p(y, z)$ refers to the pressure as a function of y and z in one of the three regions, where z is between 0 and H. All variables are independent of x.

(i) What is $p(z)$ in region I? Express your answer in terms of ρ_w, g, H, and z.

(ii) What is $p(y, z)$ in region II? Express your answer in terms of ρ_w, ρ_c, g, H, L, y, and z.

(iii) What is $p(z)$ in region III? Express your answer in terms of ρ_c, g, H, and z.

(iv) What is the geostrophic wind u_g in region I?

(v) What is the geostrophic wind u_g in region II? Express your answer in terms of ρ_w, ρ_c, g, H, L, and z.

(vi) Is there a singularity in geostrophic relative vorticity along the front? Discuss with respect to the definition of vorticity, and your answers to the previous two questions.

2. Suppose you have a layer of fluid of some constant depth h. The fluid has a "cold sector" and a "warm sector", separated by a straight sloping boundary characterized by a density discontinuity. Within each sector, the density of the fluid is constant. The fluid is gravitationally stable and the pressure at each point in the fluid is equal to the hydrostatic pressure. Let ρ_w refer to the the density of the fluid in the warm sector, and ρ_c refer to the density of the fluid in the cold sector. Assume that the front extends for a distance L in the horizontal (e.g. so that the slope of the front equals h/L).

(i) In what part(s) of the fluid will the pressure gradient acceleration be non-zero? Be as specific as possible and use a diagram of the frontal region to help you be specific.

The only region where the PGA is non-zero is under the front.

(ii) What is the surface pressure in the warm sector, away from the front? Express in terms of ρ_w, g, and h.

$\rho_w g h$

(iii) In the part of the fluid where the pressure gradient acceleration is non-zero, express the magnitude of this acceleration in terms of h, L, g, ρ_w, and ρ_c. In which direction does the acceleration point?

$(\rho_c - \rho_w)/\rho_c \times gh/L$. *This is fairly easy to derive, but can be thought of a "buoyancy" acceleration times the slope of the frontal boundary.*

(iv) Assume the fluid is in geostrophic balance. In what part of the fluid is the geostrophic wind non-zero, in what direction does it point, and what is its magnitude?

The PGA points from cold to warm. Therefore, the CA must point from warm to cold. This would require a geostrophic flow parallel to the front. Looking at the front from the warm sector, you need a flow going to your left. The flow would be non-zero where the PGA was non-zero, i.e. directly under the front only. For example, suppose you are in the warm sector. Usually there is warm direction toward the north-east in advance of the cold front. The geostrophic wind in the frontal region would point toward the south-west, and at least partially the warm advection. Of course, the real wind change would be a combination of the wind changes due to the frontal PGA and the synoptic PGA (i.e. configuration of surface highs and lows), and friction (important near the surface).

============== Chapter 7: Q Vectors and Frontogenesis ===========

1. The wind configuration given below is called Pure Geostrophic Deformation. The solid lines indicate the streamlines of the geostrophic flow. The dashed lines indicate temperature contours.

(i) Indicate the direction of V_g at locations A, B, and C.

(ii) Indicate the direction of $\partial V_g/\partial s$ at locations D and E (must know the direction of positive s in this geometry).

(iii) Indicate the direction of the $\mathbf{Q}$ vector at locations D and E.

(iv) Assume that the $\mathbf{Q}$ vector goes to zero at large distances from the axis of dilation. Indicate the general regions where you would expect $\nabla \cdot \mathbf{Q} > 0$, and where you would expect $\nabla \cdot \mathbf{Q} < 0$.

(v) Would you expect a direct or indirect circulation along the frontal boundary? Explain with reference to an equation.

(vi) Is the horizontal circulation is frontogenetic or frontolytic?

(vii) Is the induced vertical circulation is frontogenetic or frontolytic? Explain.

2.

(i) Would you expect the configuration of $\mathbf{Q}$ vectors and isentropes above to be frontogenetic or frontolytic? Explain your answer.

See Figure 7.13 and discussion. I probably should have said "horizontally frontogenetic or frontolytic", here, though this is what would ordinarily be implied. Some students discussed the induced vertical circulation, which would be direct and therefore vertically frontolytic. I gave 3.5 of 4 points for this.

(ii) Would you expect an induced direct or indirect vertical circulation. Explain.

3. The figure on the following page shows isotherms and lines of constant geopotential height.

(i) Indicate the direction and relative magnitude of $\partial \mathbf{V}_g/\partial s$ at locations A and C.

$\mathbf{V}_g$ *decreases in the positive s direction (to the right). The $\partial \mathbf{V}_g/\partial s$ vectors therefore point down, in the direction of the change in $\mathbf{V}_g$. It is important that the vector at A be larger than the vector at C, since the geopotential height gradient at A is larger.*

(ii) Indicate the direction and relative magnitudes of $\mathbf{Q}$ at locations A and C.

The Q vector points left, with the Q vector at A larger, where $\partial \mathbf{V}_g/\partial s$ is larger.

(iii) Would you expect $\nabla \cdot \mathbf{Q}$ to be positive or negative at location B?

Divergent, so $\nabla \cdot \mathbf{Q} > 0$.

(iv) Would you expect upward or downward vertical motion at location B? Explain with reference to the equation given in question 1.

downward motion. Explanation is in (ii).

(v) Would you expect the horizontal geostrophic flow to be frontogenetic or frontolytic at A and C? Explain with reference to a specific equation.

The $\mathbf{Q}$ *vector points parallel to the temperature contours so the dot product of* $\mathbf{Q}$ *and the temperature gradient is zero. So neither frontogenetic, or frontolytic. Relevant equation is* $\Im_{2D} = (1/f\gamma)\,\mathbf{Q} \cdot \nabla\theta/|\nabla\theta|$

4. Draw a configuration of isotherms and lines of constant geopotential height where $\mathbf{Q} = 0$. Explain why $\mathbf{Q} = 0$ in this configuration.

The simplest answer is any configuration where $\mathbf{V}_g$ *is constant.*

2. The diagram below shows lines of constant geopotential height (solid lines) and lines of temperature (dashed lines). You can assume that the magnitude of the temperature gradient vector is constant. The geopotential height contours are closer togethor to the right, as indicated. Answer the following questions, using the natural coordinate expression for the $\mathbf{Q}$ vector discussed in class. (Diagram has constant North-South T gradient, and GPHT lines closer togethor to the right).

(i) At point B, indicate the direction of the **s** unit vector. (2 points)

(ii) At point B, indicate the direction of the **n** unit vector. (2 points)

(iii) At points A and C, show the geostrophic wind vectors $\mathbf{V_g}$, using appropriate relative magnitudes. (4 points)

(iv) Show the $\partial \mathbf{V}_g/\partial s$ vector at point B. (4 points)

(v) Show the $\mathbf{Q}$ vector at point B. (4 points)

(vi) Assume that the $\mathbf{Q}$ vector goes to zero at the left of the diagram, as shown. In this case, would you expect upward or downward motion at point A? Explain with reference to a specific equation. (4 points)

========= Chapter 7: Inertial Instability ====================

1. What is the condition under which the absolute geostrophic momentum M_g is conserved in the atmosphere? (Assume frictionless adiabatic flow.).

No torque from a zonal PGA.

2. Suppose there was an easterly jet in the upper troposphere (i.e. zonal wind from the east).

(i) Would you expect the poleward side of the jet to have positive or negative relative vorticity? Explain using a relevant equation.

(ii) If the jet were to experience inertial instability, which side would be more likely: the poleward or equatorward side? Explain using a relevant equation.

3. We discussed in class a location where there is an easterly jet in the lower troposphere (i.e. prevailing wind from the east).

(i) Where is this easterly jet and what is its origin?

This jet is between equatorial west Africa and the Sahara (an area called the Sahel). It occurs because heating of the lower troposphere over the Sahara reverses the usual meridional temperature gradient (i.e. starting from the equator in the lower troposphere, gets warmer as you go north). It should be noted that although concepts like thermal wind and geostrophic balance are less valid in the tropics where the Coriolis acceleration is small, atmospheric scientists still often apply them and these types of explanations still often work, at least in quasi-steady situations.

(ii) Suppose this jet were to experience inertial instability. Would you expect this instability to first occur on the poleward or equatorward side? Explain.

As you through the jet in the northward direction, the momentum would initially decrease on the equatorial side since $\partial u/\partial y < 0$. Once you hit the peak easterlies (westward wind) at the jet core, and proceed poleward, you would have $\partial u/\partial y < 0$ and the momentum increasing toward the pole. This is the side where inertial instability would most likely occur.

4. In the derivation of the criteria for inertial instability, we neglected to include the variation of f with latitude (also known as the β effect). Suppose we had included this variation. Would it make inertial instability more or less likely? Explain. Assume that the initial wind configuration is purely zonal and geostrophic, that the air parcel is undergoing a northward displacement from its original latitude, and that the parcel is in the Northern Hemisphere. A diagram would help.

For a northward displacement, the Coriolis acceleration supplies the restoring force to the initial latitude. This would be larger if the β effect is considered, since it would include the increase of f toward the pole. Therefore inertial instability (at least for poleward displacements) would be less likely.

5. (i) A parcel is originally located at point P in the figure below. It is subjected to hypothetical displacements in the directions indicated by arrows 1, 2, 3, and 4. For each of each of these directions, discuss whether you would expect a net parcel acceleration away from point P, or toward point P. For each case, refer to expected accelerations associated with changes in θ and M.

See question 7.7 from the text.

(ii) Is this flow inertially stable? Why or why not?

(iii) Within clouds, the evaluation of inertial stability is made with respect to the variation of M on θ_e surfaces rather than θ surfaces. Explain.

2. (i) Assume that a flow is purely zonal and in geostrophic balance. What is the criteria for inertial instability in this type of flow? (Hint: involves M_g)

(ii) In general, would you expect inertial instability to more likely develop on the polar or equatorward side of a jet stream? Explain.

3. On some pressure surface, the geopotential varies in the y (north - south) direction as given by

$$\Phi(y) = \Phi_0 + af^2y^2,$$

where Φ_0 and a are constants, and the Coriolis parameter f can be treated as a constant.

(i) Evaluate the geostrophic wind vector $\mathbf{V_g}$. (8 points)

(ii) For what values of the parameter a is this geopotential height configuration inertially unstable? Explain. (18 points)

======= **Chapter 8: QG Height Tendency Equation** ========

1. The formula below is known as the Quasi Geostrophic Geopotential Tendency Equation. Note that $\chi = \partial\Phi/\partial t$.

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi = -f_0\mathbf{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) - \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\mathbf{V}_g \cdot \nabla\left(\frac{\partial\Phi}{\partial p}\right)\right]$$

(i) Suppose that a surface low is located between an upper level trough and an upper level ridge and has started to generate a low-level cyclonic circulation. There is a baroclinic zone near the surface. Would you expect warm advection or cold advection at low levels to the east of the surface low? Explain.

(ii) Consider the term:

$$\mathbf{V}_g \cdot \nabla\left(\partial\Phi/\partial p\right)$$

Would you expect this term to be positive or negative near the surface to the east of the surface low? Explain in detail mathematically starting with your answer in (i).

(iii) Consider the term:

$$\frac{\partial}{\partial p}\left[\mathbf{V}_g \cdot \nabla\left(\frac{\partial\Phi}{\partial p}\right)\right]$$

Would you expect this term to be positive or negative in the lower troposphere to the east of the surface low? Explain in mathematical detail starting with your answer to (ii).

(iv) Suppose we ignore the first term on the right of the quasi geostrophic geopotential tendency equation and consider the effect of the second temperature advection term alone. Would you expect

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi$$

to be positive or negative to the east of the surface low? Explain in detail mathematically.

(v) Would expect χ to be positive or negative in the upper troposphere to the right of the surface low? Explain in detail.

(vi) Use a diagram to illustrate the effect of this term on the shape of the geopotential lines in the upper troposphere between the trough and the ridge.

(vii) Would you expect the changes in the geopotential height contours shown in your answer to (vi) to increase or decrease the vorticity gradient above the developing low? Explain.

2. There is a developing low at the surface in a baroclinic zone located half way between an upper level trough and an upper level ridge.

(i) In the mid-troposphere, to the right of the surface low, would you expect the second term on the right of the QG geopotential tendency equation to be positive or negative? Explain in detail. Discuss the vertical variation, and signs in the upper and lower troposphere, of the relevant quantities. (In future: break down this question into parts, about 4 steps.)

page 248. Briefly, there is warm advection near the surface to the right of the low. The term given in 2. is positive. But it decreases with height, since T advection weaker in the upper troposphere. Or it increases toward the surface, so in (8.4) contributes to a negative RHS of (8.4).

(ii) Suppose the second term on the right hand side of the QG height tendency equation dominates. For the situation described above, would expect χ to be positive or negative in the upper tropospheric region to the right of the surface low?

As usual if the 3d Laplacian of a quantity is negative, we consider that quantity to be positive, so $\chi > 0$.

(iii) How would this height tendency effect the vertical motion associated with the surface low pressure system? Explain in detail.

Increased downstream ridging gives rise to anticyclonic curvature, so increased PVA over low, which increases upward motion. Could also talk about wavelength shortening.

==================== **Chapter 8: TROWAL** ==============

1. The figure below has been copied from Figure 8.18(a) from the text. The solid lines refer to contours of equivalent potential temperature θ_e, and shaded regions to upward motion. It is a cross-section of a warm occluded zone.

(i) Indicate the position of the TROWAL with an T.

(ii) Indicate the position of the warm occluded front at the surface with a WOC.

(iii) Show the position of the cold front aloft using a line of the standard triangles.

(iv) Show the warm front aloft using the standard symbol of open half circles.

(v) Indicate the region of most likely convective instability.

the height interval behind the cold front where θ_e decreases with height, a precondition for layer convective instability.

(vi) Speculate on the reasons why θ_e appears well mixed (change to relatively uniform) in the region of strongest upward motion.

Strong upward motion and mixing will tend to make θ_e homogeneous.

2. Define the TROWAL.

intersection of two cold fronts with lifted warm sector, can also give the acronym, and describe.

3. Why is the TROWAL a useful forecasting tool?

Projection on to the surface useful guide for surface precipitation

=============== **Chapter 8: Cyclone Structure** ===============

1. The satellite image in the next page shows a mature cyclonic storm. Identify three interesting aspects of this image, and briefly discuss its physical origin.

2. The diagram below shows a vertical cross-section of a developing mid-latitude synoptic system along the east-west direction. The positions of the upper level trough (L) and ridge (R) have been labeled.

(i) Indicate the likely position of the developing low pressure system at the surface. (2 points)

(ii) Indicate the likely position of high pressure at the surface. (2 points)

(iii) Indicate with a dashed line, and label, the warm core axes.

At the surface, warm air lies at the low or slightly east, as a response to warm advection on the eastern side of the surface low. However, warm air axis must tilt to the right to build the upper ridge.

(iv) Indicate with a dashed line, and label, the cold core axes.

low-level cold air advection occurs between the surface high and low, with the result that get coldest temperatures near low at the surface (Note again: the temperature advection and temperature itself must be out of phase, for a train of highs and lows moving to the east!). Cold air axis must go toward the trough to lower GPHT at the trough.

(v) Indicate the most likely location of upper level divergence.

(vi) Indicate the most likely location of upper level convergence.

(vii) Indicate the most likely location of mid-level ascent with an up arrow.

Upward motion occurs due to upper level divergence between the trough and ridge (PVA or ageostrophic divergence, pick your explanation). Should also say need upward motion ahead of developing surface low is needed to drive down surface pressure. Note that the surface pressure and vertical motion must be out of phase.

(viii) Indicate the most likely location of mid-level descent with a down arrow.
Downward motion occurs ahead of the developing high, or between the ridge and trough where you tend to have ageostrophic convergence.

(ix) Indicate the approximate location of the trough axis.

(x) Indicate the approximate location of low-level warm advection.

(xi) Indicate the approximate location of low-level cold advection.

(xii) Indicate the approximate location of strongest upper level positive vorticity advection.
2. The diagram below shows a vertical cross-section of a developing mid-latitude synoptic system along the east-west direction. The positions of the upper level trough (L) and ridge (R) have been labeled.

(i) Indicate with a dashed line, and label, the warm core axes. (1 point)

(ii) Indicate with a dashed line, and label, the cold core axes. (1 point)

(iii) Indicate using DIV the most likely location of upper level divergence. (1 point)

(iv) Indicate using CONV the most likely location of upper level convergence. (1 point)

(v) Indicate the most likely location of mid-level ascent with an up arrow. (1 point)

(vi) Indicate the most likely location of mid-level descent with a down arrow. (1 point)

(vii) Indicate with a T the approximate location of the trough axis. (1 point)

(viii) Indicate using WA the approximate location of low-level warm advection. (1 point)

(ix) Indicate using CA the approximate location of low-level cold advection. (1 point)

(x) Indicate using VA the approximate location of strongest upper level positive vorticity advection. (1 point)

======= Chapter 8: Cyclogenesis/Cyclolysis ================

1. What are the two *most important* factors contributing to the origin of *most* cyclogenesis events in mid-latitudes over land?

Advection of vorticity (or PV) anomaly over baroclinic zone.

2. What is the most important factor contributing to the cyclolysis (decay) of a surface cyclone. Explain.

Loss of contact with upper tropospheric divergence, i.e. "valve" required to remove air associated with frictionally induced low level convergence. Could also talk about loss of phase relationship with upper level trough, so don't have PVA above the surface low.

3. (i) Specify two factors that are believed to be especially important in contributing to explosive cyclogenesis events.

The key word here is "explosive": enhanced water vapor and reduced lower tropospheric stability

(ii) How does the geographic distribution of explosive cyclogenesis events support the importance of these two factors?

Often occur over western boundary currents (Gulf Stream and Kirohisho) in winter where water is warm (so reduced lower tropospheric stability), and large moisture source.

4. Mid-latitude synoptic storms which contain more moisture may intensify more quickly. Briefly discuss a positive feedback involving water vapor whereby this may occur.

=== Chapter 9: Upper/Lower PV Anomaly : Obtaining Wind Patterns ====

1. There is a low-level warm anomaly near the surface. Assume that on the 500 hPa pressure surface, the potential temperature (PT) contours can be considered flat, and that $v_g = 0$. You can also assume that the wind is well approximated by the geostrophic wind.

(i) Draw a vertical cross-section of the PT contours in the vicinity of the warm anomaly between 500 hPa and the surface. The horizontal axis should be the east-west direction (with east to the right of the page).

Figure 9.7

(ii) What is the sign of dT/dx (positive or negative) near the surface on the western side of the warm anomaly?

(iii) What is the sign of dT/dx (positive or negative) near the surface on the eastern side of the warm anomaly?

(iv) What is the sign of dv_g/dp (positive or negative) near the surface on the western side of the warm anomaly? Explain using a relevant equation.

Invoke the boundary condition at 500 hPa, use the sign of $\partial T/\partial x$, and use the formula below.

$$p\frac{\partial v_g}{\partial p} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p$$

(v) What is the sign of dv_g/dp (positive or negative) near the surface on the eastern side of the warm anomaly? Explain using a relevant equation.

(vi) Would you expect northward ($v_g > 0$) or southward ($v_g < 0$) winds on the western side of the warm anomaly, near the surface? Explain.

(vii) Would you expect northward ($v_g > 0$) or southward ($v_g < 0$) winds on the eastern side of the warm anomaly, near the surface? Explain.

(viii) Would you expect low-level warm anomalies to be characterized by positive or negative relative vorticity? Explain using an equation.

2. There is a positive PV anomaly in the upper troposphere. The PV anomaly is also a positive vorticity anomaly. You can assume that the geostrophic wind is small in the lower troposphere.

(i) Below the upper level PV anomaly, on the western side, would you expect $\partial v_g/\partial p$ to be positive or negative? Explain your choice mathematically. (4 points)

(ii) Below the upper level PV anomaly, on the eastern side, would you expect $\partial v_g/\partial p$ to be positive or negative? Explain your choice mathematically. (4 points)

(iii) Use a thermal wind equation to argue whether $\partial T/\partial x$ is positive or negative below the PV anomaly on the western side. (4 points)

(iv) Use a thermal wind equation to argue whether $\partial T/\partial x$ is positive or negative below the PV anomaly on the eastern side. (4 points)

(v) Given your previous answers, draw a vertical cross-section of the displacement of θ contours associated with a large upper level positive PV anomaly in the upper troposphere. The vertical axis should extend from the surface to the lower stratosphere. The horizontal axis should be in the east-west direction, with east to the right of the page. (6 points)

===== Chapter 9: Potential Vorticity (PV) and Precipitation =======

1. (i) The figure below is taken from Figure 9.14 of the text. The dark shaded regions correspond to regions of precipitation along a cold front. Indicate on the figure the lower tropospheric wind anomalies that would be generated by the heating anomalies associated with this deep convective precipitation.

(ii) How might these lower tropospheric wind anomalies affect the development of a synoptic low?

2. The figures below are 14 (a) and (c) from the Appenzeller paper (JGR, Volume 101, 1996).

(i) Indicate in the figure (in a or c) the likely region of deep convection, and explain your choice.

(ii) Why might deep convection occur in association with an upper level PV anomaly?

(iii) In (a), indicate the region where you would expect the diabatic heating from the deep convection to produce PV, and where you would expect it do destroy PV.

(iv) Speculate on how a potential vorticity tendency from deep convection might alter the local height of the tropopause (2 PVU contour).

3. From a PV perspective, how does latent heat release affect the development of a synoptic low pressure system (briefly). Use a diagram to help your explanation.

Looking for Figure 9.12 and associated explanation.

4. Precipitation is giving rise to a rate of diabatic heating that is zero at the surface and the tropopause, and peaks in the mid-troposphere. Plot the vertical profile of the potential vorticity tendency that would be associated with this heating profile. The vertical axis should be height, and indicate the height of the tropopause, and the height of maximum heating, in your figure.

Plot should show $dPV/dt = 0$ at height of maximum heating, zero at the surface and tropopause (since the heating is zero there), negative in the upper troposphere, and positive in the lower troposphere.

5. The figure below shows a precipitating anvil cloud whose base is in the mid-troposphere. It is located in the Northern Hemisphere. Some of the precipitation falling from the cloud evaporates before it hits the ground. Answer the following questions, taking into consideration the diabatic effects associated with precipitation formation and evaporation within and below the cloud.

This is a question where you have a heat source in the upper troposphere, and a heat sink (cooling) in the lower troposphere. This would increase the stability of the middle troposphere, and increase the PV and relative vorticity at that level. The other points would have referred to the surface and the top of the troposphere. The answer would go by the vertical gradient in heating. If positive, increase stability, PV, etc. If not, decrease.

(i) At each of the points A, B, and C: would you expect the stability to be increasing or decreasing?

(ii) At each of the points A, B, and C: would you expect the potential vorticity to be increasing or decreasing? Explain.

(iii) At which of the three points A, B, or C would you expect the strongest rate of increase in counter-clockwise rotation?

9. (i) The rain rate at the surface is 50 mm/day. Assume that all of the latent heat from this rain formation is evenly distributed between 700 hPa and 300 hPa to generate a constant heating rate Q. What is Q? Express your answer in J/kg s. You can assume that the latent heat of vaporization is $l_{v0} = 2.5 \times 10^6$ J/kg. (4 points)

(ii) During the above heating, assume that air parcels remain fixed at pressure levels, and that initially $T = 270$ K at $p = 500$ hPa. Calculate the potential temperature at 500 hPa before and after the heating. Assume the heating lasts for one day. (4 points)

(iii) Assume that the potential temperature tendency at 200 hPa is zero. Estimate the mean PV tendency between 500 hPa and 200 hPa while the rainfall is occurring. Assume that the relative vorticity is always zero at both pressure levels. Assume a latitude of 45 degrees. (4 points)

========= Chapter 9: Potential Vorticity (PV): Other Questions =====

1. Does a low level warm anomaly tend to propagate to the west or the east? Explain using a diagram in the horizontal plane (near the surface) showing the displacement of PT contours associated with the warm anomaly, and use it to help explain the direction of propagation.

Figure 9.9 and associated explanation.

2. Some upper level PV anomalies are better at contributing to the development of cyclogenesis than others. What is one characteristic of an upper level PV anomaly that helps determine its effectiveness, and explain why this characteristic is important.

larger horizontal spatial scale, so larger penetration depth, so capable of triggering temperature advection at the surface.

3. Suppose there is a negative PV anomaly in the mid-troposphere. The PT surface going through the center of the anomaly is flat. Show a vertical cross-section of how you would expect the PT contours to deform around the negative PV anomaly.

Figure 9.5(a)

============== **Waves** ===========

3. The diagram below shows a shallow water wave in one dimension propagating to the right. Along the dashed line, identify the following using the appropriate letter (i.e. write an a at the location of maximum pressure). Let h refer to the depth of the fluid. Ignore the Coriolis acceleration. Let $dp_{nh} = p - p_{hyd}$ refer to the deviation of the actual pressure from the hydrostatic pressure. (This figure shows a trough, peak, trough pattern. The dashed line is a level line in the interior of the fluid.)

(a) maximum pressure (1 point)
(b) most negative $\partial p/\partial x$ (1 points)
(c) maximum $\partial u/\partial t$ (1 points)
(d) maximum u (1 point)
(e) maximum du/dx (1 point)
(f) maximum convergence (1 point)
(g) maximum dh/dt (1 point)
(h) minimum dh/dt (1 point)
(i) maximum dp_{nh} (1 point)
(j) minimum dp_{nh} (1 point)

============== **Extra Topic: Rossby and Kelvin Waves** ==========

1. (i) What does it mean to say a wave is dispersive?

speed depends on wavelength.

(ii) Give an example in atmospheric science of a dispersive wave.

Rossby waves

(iii) Are sound waves dispersive or non-dispersive? Explain your answer using a fact from everyday experience.

No, since we can hear music. If notes of different pitch traveled at different speeds, then the "sound" of a musical instrument change with distance.

2. Are Rossby waves dispersive or non-dispersive? Explain.

dispersive, since speed depends on wavelength

3. The main assumption in our derivation of the Rossby wave dispersion relationship was that the absolute (total) vorticity is conserved following the air parcel motion:

$$\frac{d\eta}{dt} = 0.$$

(i) Starting from this assumption, the definition of absolute vorticity, the definition of β, the definition of total derivative, derive the following differential equation for the relative vorticity ξ. Assume there is no vertical wind.

$$(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y})\xi + \beta v = 0$$

(ii) Assume that the total horizontal wind can be expressed as $u = \bar{u} + u'$ and $v = v'$, where $\bar{u}$ is a constant and u' and v' can be considered small. Using reasonable approximations, and with ξ' defined as usual in terms of u' and v', Derive the following equation.

$$(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x})\xi' + \beta v' = 0$$

(iii) The next step in the derivation of the Rossby wave dispersion relation is to set $u' = -\partial\psi'/\partial y$ and $v' = \partial\psi'/\partial x$. Show that with this substitution, one can write.

$$(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x})\nabla^2\psi' + \beta\frac{\partial\psi'}{\partial x} = 0$$

(iv) The assumption that the absolute vorticity is conserved following the flow requires that the horizontal flow be non-divergent.

$$\nabla \cdot \mathbf{V} = 0$$

The total flow refers to the sum of the background and wave flow. Show that if the wave wind field is expressed in terms of a stream function as given above, that the total flow is, indeed, non-divergent.
(iv) Assume that the perturbation stream function can be expressed in terms of an amplitude ψ, wavenumber k, and frequency ν as shown below. The phase speed is defined as $c = \nu/k$. Derive an expression for c in terms of β, $\overline{u}$, and k. (14 points)

$$\psi' = Re\left[\psi e^{\left(i(kx-\nu t)\right)}\right]$$

10. (i) We discussed in class the approximations involved in using the following three governing equations to derive the dispersion relationship for Kelvin waves. In these equations, u and h refer to the zonal wind and height perturbations associated with the Kelvin wave, h_e to the mean depth of the fluid, and $\beta = \partial f/\partial y$, evaluated at the equator.

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}$$

$$\beta y u = -g\frac{\partial h}{\partial y}$$

$$\frac{\partial h}{\partial t} + h_e \frac{\partial u}{\partial x} = 0$$

(i) Specify three important, independent assumptions made in arriving at these three governing equations. (3 points)
(ii) Assume wavelike solutions of the form $u = \hat{u}(y)e^{i(kx-\nu t)}$ and $h = \hat{h}(y)e^{i(kx-\nu t)}$. Re-express the three governing equations given above in terms of the wave amplitude or "hat" variables. (3 points)

(iii) Show that $c = \sqrt{gh_e}$ for Kelvin waves. (4 points)
(iv) Show that $\hat{u}(y) = u_0 e^{-\beta y^2/2c}$, where u_0 is the amplitude of the Kelvin wave at the equator. (4 points)
(v) Equatorially trapped Kelvin waves must propagate eastward. Explain. (1 point)
(vi) In the $x - y$ (longitude-latitude) plane draw the pattern of perturbation height $h(x, y)$ associated with a Kelvin wave. Indicate the center of the low by an "L", and the center of the high by an "H". Make sure you show one complete wavelength, and show both sides of the equator. Show on the diagram enough arrows indicating the perturbation zonal velocity u to characterize the variation in u associated with a Kelvin wave. Finally, show on your diagram a region of maximum convergence, and a region of maximum divergence. (4 points)
(vii) Is the force balance of the u velocity of a Kelvin wave in the zonal direction in geostrophic balance? Explain why or why not. (1 point)
(vii) Is the force balance of the v velocity of a Kelvin wave in the zonal direction in geostrophic balance? Explain why or why not. (1 point)