Math 4020/5020 - Assignment 2 - Winter 2012.

Deadline: Monday, January 23th 11:35 am.

Late assignments (5 minutes to two days) will be penalized 10%.

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1. Use the ϵ-δ definition of continuity to prove that the function $f(z) = \overline{z} + z^3$ is continuous on $\mathbb{C}$. Find the largest subset of $\mathbb{C}$ on which f is differentiable. Reference any theorem that you use.

2. Prove that the function $f(z) = z\mathrm{Re}(z)$ is differentiable only at the point $z = 0$, and find $f'(0)$.

3. Prove that the Cauchy-Riemann equations are satisfied for the function $f(x + iy) = \sqrt{xy}$ at the point $z_0 = 0$, but the derivative of f at $z_0 = 0$ does not exist. Explain why this does not contradict the theorem on the sufficient condition for differentiability.

4. Let $D, \Omega \subseteq \mathbb{C}$ be domains. Show (with an $\epsilon - \delta$ proof) that if f is continuous on D, and g is continuous on Ω, and $f(D) \subseteq \Omega$, then the composition function $g \circ f$ is continuous on D as well.

5. [**Bonus for MATH4020**] Let $D, \Omega \subseteq \mathbb{C}$ be domains. Prove that if f is analytic on D, and g is analytic on Ω, and $f(D) \subseteq \Omega$, then the composition function $g \circ f$ is analytic on D, and the chain rule holds:

$$(g \circ f)'(z) = g'(f(z))f'(z) \quad \forall z \in D.$$

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems, and prove your claims. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.