Math 4020 - Assignment 3 - Winter 2011.

Deadline: Friday, February 10th 11:35 am.

Late assignments (5 minutes to two days) will be penalized 10%.

---

1. Let γ, γ_1 and γ_2 be piecewise smooth curves. Let $f : \mathbb{C} \to \mathbb{C}$ be continuous on the curves γ, γ_1 and γ_2. Prove that

   (i) $\int_\gamma f(z)dz = -\int_{-\gamma} f(z)dz$.

   (ii) $\int_{\gamma_1+\gamma_2} f(z)dz = \int_{\gamma_1} f(z)dz + \int_{\gamma_2} f(z)dz$.

   (iii) If γ is a closed curve, and γ_c is the curve obtained from γ by shift of parameter, then $\int_\gamma f(x)dx = \int_{\gamma_c} f(x)dx$.

2. Let γ_1 and γ_2 be piecewise smooth curves.

   (i) Prove that $L(\gamma_1) = L(-\gamma_1)$.

   (ii) Prove that $L(\gamma_1 + \gamma_2) = L(\gamma_1) + L(\gamma_2)$.

3. Let f be a complex function which is analytic on a domain $\mathcal{U}$. Suppose $f'(z)$ is continuous on $\mathcal{U}$. Prove that $\int_\gamma f(z)f'(z) = 0$, for every piecewise smooth closed curve γ that lies entirely in $\mathcal{U}$.

4. Let f be an analytic function on the open disc $b_1(0)$. Assume that f' is continuous and bounded on $b_1(0)$. Prove that there exists $C > 0$ such that

$$|f(z_1) - f(z_2)| \leq C|z_1 - z_2| \quad \forall z_1, z_2 \in b_1(0).$$

5. Compute $\int_\gamma \frac{1}{z}dz$, where γ is the circle of radius r centred at the origin and oriented counter clockwise.
   Note: Do not use any theorems that has not been covered in the lectures.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.