Math 4020/5020 - Assignment 4 - Winter 2012.

Deadline: Monday, February 27th 11:35 am.

Late assignments (5 minutes to two days) will be penalized 10%.

Note: You are only allowed to use the results covered in the lectures.

1. Compute the following integrals.

   (i) $\int_\gamma \frac{e^{z^2}-z^3}{z+i}dz$, where γ is a piecewise smooth simple closed curve in the upper half-plane oriented positively.

   (ii) $\int_\gamma \frac{z^2 e^{z^3}}{z^2+1}dz$, where γ is a piecewise smooth simple closed curve oriented positively.

   (iii) $\int_\gamma \frac{\bar{z}}{z^2}dz$, where γ is the circle of radius 1 centered at the origin and oriented positively.

   (iv) $\int_\gamma \frac{|z|e^z}{2z-1}dz$, where γ is the circle of radius 1 centered at the origin and oriented clockwise.

2. Let f be a function analytic on the open disc $b_1(0)$ (i.e. the open disc centered at the origin of radius 1). Prove that if $f(b_1(0)) \subseteq b_1(0)$ then $|f'(0)| \leq 1$.

3. Let f be an entire function. Suppose that there exists an integer $n > 0$ such that the nth derivative of f, $f^{(n)}$, is identically zero on $\mathbb{C}$. Show that f must be a polynomial.
   Hint: Use mathematical induction.

4. Let f be an entire function. Suppose that there exists $n \in \mathbb{N}$ and $K > 0$ such that $|f(z)| < K|z|^n$ for every z in $\mathbb{C}$. Prove that f has to be a polynomial.
   Hint: Use Question 3.

5. Let f be an entire function (i.e. f is analytic on $\mathbb{C}$). Suppose that there exists a constant $M > 0$ such that $|f(z)| \leq M$ for every z in $\mathbb{C}$. Prove that f is a constant function.
   Hint: You may use Cauchy integral formula.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.