Math 4020 - Assignment 5 - Winter 2012.

Deadline: Monday, March 19th 11:35 am.

Late assignments (5 minutes to two days) will be penalized 10%.

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1. Let $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ be a power series of radius of convergence $R>0$. Prove that the power series converges uniformly on $b_r(z_0)$ for every $r<R$.

2. "Analytic continuation is unique". Let D_1 and D_2 be two domains such that $D_1 \cap D_2 \neq \emptyset$ is a domain as well. Suppose f is an analytic function on D_1. A function g is called an analytic continuation of g into D_2 if f is analytic on D_2 and $f(z)=g(z)$ for $z \in D_1 \cap D_2$. Prove that for D_1, D_2 and f as above, there is a unique analytic continuation.

3. Find the Taylor series expansion of the following functions. In each case, find the domain in which the expansion converges to the function.

   - $\cos(z) := \frac{1}{2}(e^{iz}+e^{-iz})$ about $z=0$.
   - $\sin(z) := \frac{1}{2i}(e^{iz}-e^{-iz})$ about $z=0$.
   - $\frac{2z}{z^2+9}$ about $z=0$.
   - $\sin(z^2)$ about $z=0$.
   - $z\cos(z)$ about $z=\frac{\pi}{2}$.

4. Find the Laurent series expansion of the function $f(z)=\frac{1}{z^5(z+2)}$ about the origin in all the possible domains.

5. Prove that if f is analytic at z_0 and $f(z_0)=f'(z_0)=\ldots=f^{(k)}(z_0)=0$, then the function g defined below is analytic at z_0.

$$g(z)=\begin{cases} \frac{f(z)}{(z-z_0)^{k+1}} & \text{if } z \neq z_0 \\ \frac{f^{(k+1)}(z_0)}{(k+1)!} & \text{if } z = z_0 \end{cases}$$

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.