Math 4020/5020 - Assignment 7 - Winter 2012.

Deadline: April 10th.

Late assignments are not accepted.

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1. Determine the number of zeros, counting multiplicities, of the following polynomials in the specified domains.

   (i) $z^6 - 5z^4 + z^3 - 2z$, where D is the interior of the circle $|z| = 1$.

   (ii) $2z^4 - 3z^3 - z + 9$, where D is the interior of the circle $|z| = 1$.

   (iii) $z^4 + 3z^3 + 6$, where D is the interior of the circle $|z| = 2$.

   (iv) $2z^5 - 6z^2 + z + 1$, where D is the annulus $1 \leq |z| < 2$.

2. Let C be the circle of radius 1 centered at the origin. Suppose that f is analytic on and inside C, and satisfies $|f(z)| < 1$ for every $z \in C$. Show that the equation $f(z) = z^4$ has exactly four solutions (counting multiplicities) inside C.

3. Prove that if $1 < \lambda < \infty$, the function

   $$f_\lambda(z) = z + \lambda - e^z$$

   has only one zero in the half-plane $\{z \in \mathbb{C} : \mathrm{Re}(z) < 0\}$. Moreover, this zero lies on the real axis.

4. Evaluate the following integrals using Cauchy's integral formula.

   (i) $\int_0^{2\pi} e^{e^{i\theta}} d\theta$.

   (ii) $\int_0^{2\pi} e^{e^{i\theta} - i\theta} d\theta$.

5. Does there exist a non-constant entire function f that satisfies

   $$f(z) = f(z+1) \quad \text{and} \quad f(z) = f(z+i),$$

   for all $z \in \mathbb{C}$? Support your answer.

Comments: The submitted solutions must be tidy and legible. You are to provide full solutions to the problems. You are allowed, and encouraged to collaborate with your classmates, but the write-ups should be done individually, without access to the papers of fellow students. Copying assignments or tests from any source, completely or partially, allowing others to copy your work, will not be tolerated.