Pharm 3011 - Fall 2019 - Assignment 3 Solutions
out of 40 points

1. A logistic regression of "toxemia" (1=yes, 0=no) on mother's age was carried out. Where p_x denotes the probability of toxemia given a mother of age x, the model fit was

$$log\left(\frac{p_x}{1-p_x}\right) = \alpha + \beta x$$

leading to the following partial computer output.

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Logistic Regression Table

Predictor        Coef   SE Coef   Z
Intercept       -16.0     4.0
mother's age      0.8      0.4
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(a) What is the estimated log odds of toxemia for a mother of age 25 years?
(1) -16+.8(25) = 4

(b) What is the estimated log odds of toxemia for a mother of age 30 years?
(1) -16+.8(30) = 8

(c) What is the odds ratio for a 30 year old mother, as compared to a 25 year old mother?
the estimated log odds ratio for a 30 year old mother compared to a 25 year old mother
(3) 8-4 = 4
the estimated odds ratio for a 30 year old compared to a 25 year old mother is $e^4 \approx 54.6$. (deduct 1 point if the reciprocal of the odds ratio is calculated [ie $e^{-4} \approx .018$], or if base 10 logarithms were used)

(d) To test the hypothesis $H_0 : \beta = 0$ against the alternative $H_0 : \beta \neq 0$, what is the observed value of the test statistic?
(2) The test statistic is $Z = \frac{\hat{\beta}}{\widehat{s.e.}(\hat{\beta})}$, which in this case is $.8/.4 = 2$

(e) Construct a 90% confidence interval for β.
(3) In general, the confidence interval is
$\hat{\beta} \pm Z_{.05}\widehat{s.e.}(\hat{\beta})$, which in this case, is
$.8 \pm 1.645(.4)$, or $.8 \pm .658$, or approximately (.142, 1.458) (full marks for anything which rounds to (.14,1.46). Deduct 1 point for each error).

(f) Construct a 90% confidence interval for the log odds ratio,

$$log\left(\frac{p_{26}}{1-p_{26}}\frac{1-p_{25}}{p_{25}}\right).$$

(2) β_1 is equal to $log\left(\frac{p_{26}}{1-p_{26}}\frac{1-p_{25}}{p_{25}}\right)$, so the confidence interval for this log odds ratio is identical to the confidence interval for β_1. Give 2 points for reporting the same interval as in part (e).

(g) Transform the CI from part f to get a 90% confidence interval for the odds ratio

$$\frac{p_{26}}{1-p_{26}} \frac{1-p_{25}}{p_{25}}.$$

(2) The CI for the odds ratio has endpoints which are calculated by exponentiating the endpoints of the CI for the log odds ratio. (Give 2 points for exponentiating the endpoints of the interval in part e or part f. Deduct 1 point if log to base 10 is used). Based on my answer to part (e), the CI is $(e^{.142}, e^{1.458})$, or approximately (1.15, 4.30).

(h) Based on the CI calculated in part g, would you reject the null hypothesis in part *d* when testing at level .10? Why?

(3) Yes. From part (g), we would reject the hypothesis that the odds ratio is 1 when testing at level .1, because the 90% CI for the odds ratio does not contain 1. But the odds ratio equals 1 if and only if $\beta = 0$, so we would reject the hypothesis that $\beta = 0$ at level .10. (1 point for answering yes, 2 points for some similar explanation).

2. A second study estimated the log odds of toxemia for a 25 year old mother as -3. For this second study, what is the estimated probability of toxemia for that mother.

$$log(\frac{p}{1-p}) = -3 \rightarrow \frac{p}{1-p} = e^{-}3 \approx .0498$$

$$\frac{p}{1-p} = .0498 \rightarrow p = \frac{.0498}{1+.0498} \approx .047$$

(3)

(3 marks for something which rounds to .05. Deduct 1 point for each mistake in the arithmetic, or for using log base 10)

3. A study was carried out to compare two population means, μ_1 and μ_2. A 95% confidence interval for $\mu_1 - \mu_2$ was calculated as (-8.5, 12.9), and a 90% confidence interval was calculated as (-6.9, 10.7).

(a) When carrying out an equivalence test of $H_0 : |\mu_1 - \mu_2| \geq 15$ vs. $H_A : |\mu_1 - \mu_2| < 15$ at level .05, would you reject the null hypothesis? Why?

(6) Yes, because the 95% CI for $\mu_1 - \mu_2$ is contained in (-15, 15)

(b) Assuming that a large value of the mean is a favourable outcome, when carrying out a non-inferiority test of $H_0 : \mu_1 \leq \mu_2 - 15$, vs. $H_A : \mu_1 > \mu_2 - 15$ at level .05, would you conclude that treatment 1 (which gave mean μ_1) is noninferior to treatment 2 (which gave mean μ_2). Why?

(7) Yes, because the lower end of the 90% confidence interval for $\mu_1 - \mu_2$ is GREATER THAN -15

(3 points for correct Y/N answer, and 4 points for some similar reasoning, including reference to the 90% CI. deduct 3 points if the 95% CI was used, which is incorrect.)

(c) Assuming that a large value of the mean is a favourable outcome, when carrying out a superiority test of $H_0 : \mu_1 \leq \mu_2 + 15$, vs. $H_A : \mu_1 > \mu_2 + 15$ at level .05, , would you conclude that treatment 1 (which gave mean μ_1) is superior to treatment 2 (which gave mean μ_2). Why?

(7) No, because the lower end of the 90% confidence interval for $\mu_1 - \mu_2$ is SMALLER THAN 15

(3 points for correct Y/N answer, and 4 points for some similar reasoning, including reference to the 90% CI. deduct 3 points if the 95% CI was used, which is incorrect.)