How to determine the degrees of freedom in One-way and Two-way ANOVA?

The degrees of freedom (DF) are the number of independent pieces of information. In ANOVA analysis once the Sum of Squares (e.g., SStr, SSE) are calculated, they are divided by corresponding DF to get Mean Squares (e.g. MStr, MSE), which are the variance of the corresponding quantity. The ratio of MStr to MSE is the observed F (F_{obs}) random variable, which has an F distribution with two DFs (often called **numerator** and **denominator DF**; also called *df1* and *df2* respectively) and is used to find the *p*-value for the ANOVA test.

In a **One-way ANOVA** with a samples (i.e., treatment groups) and there are n observations in total among the a samples, the degrees of freedom are:

For total SS: $n - 1$.
For SStr: $a - 1$.
For SSE: $n - a$.
So MStr = SStr/(a-1) and MSE = SSE/(n-a).

Then F_{obs} = MStr/MSE.
p-value = $P(F_{a-1,n-a} \geq F_{obs})$.

In the forced expiratory volume example, there are three hospital groups and 60 observations in total, so $a = 3$ and $n = 60$. The DF for total SS is 59; it is 2 for SStr and 57 for SSE. Here 2 and 57 are the numerator and denominator DFs. The denominator DF (*df2*) is always associated with the SSE to get MSE.

In a **Two-way ANOVA** with factor A of a levels and factor B of b levels and each level of factor A and factor B combination has r replicates of observations, the degrees of freedom are:

For total SS: $a\ b\ r - 1$.
For SS for factor A: $a - 1$.
For SS for factor B: $b - 1$.
For SS for A and B interaction: $(a - 1)\ (b - 1)$.
For SS for Residuals (Error term): $a\ b\ (r - 1)$.

In our histamine shock example, $a = b = 2$, $r = 10$. Then the DF for total SS is 39; for SSA (SS for factor A) is 1; for SSB is 1; for SSAB (interaction) is 1; for SSE is 2 x 2 x 9 = 36.

The Mean Squares:
For factor A: MSA = SSA/$(a - 1)$.
For factor B: MSB = SSB/$(b - 1)$.
For A and B interaction: MSAB = SSAB/$((a - 1)\ (b - 1))$.
For Residuals: MSE = SSE/$(a\ b\ (r - 1))$.

The observed F ratios (F_{obs}):
For factor A: SSA/MSE.
For factor B: SSB/MSE.
For A and B interaction: SSAB/MSE.

p-value = $P(F_{df1,\, df2} \geq F_{obs})$.

Here *df1* (numerator DF) is the DF for the factors or their interaction; *df2* (denominator DF) is that for the error term, which equals $a\ b\ (r - 1)$ in the above notation.