Solution to assignmet 2

30.

a. Because order is important, we'll use $P_{8,3} = 8(7)(6) = 336$.

b. Order doesn't matter here, so we use $C_{30,6} = 593{,}775$.

c. From each group we choose 2: $\binom{8}{2} \bullet \binom{10}{2} \bullet \binom{12}{2} = 83{,}160$

d. The numerator comes from part c and the denominator from part b: $\dfrac{83{,}160}{593{,}775} = .14$

e. **We use the same denominator as in part d. We can have all zinfandel, all merlot, or all cabernet, so P(all same) = P(all z) + P(all m) + P(all c) =**

$$\frac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593{,}775} = .002$$

32.

a. $5 \times 4 \times 3 \times 4 = 240$

b. $1 \times 1 \times 3 \times 4 = 12$

c. $4 \times 3 \times 3 \times 3 = 108$

d. # with at least on Sony = total # - # with no Sony = 240 – 108 = 132

e. P(at least one Sony) = $\frac{132}{240} = .55$

P(exactly one Sony) = P(only Sony is receiver)
+ P(only Sony is CD player)
+ P(only Sony is deck)

$$= \frac{1 \times 3 \times 3 \times 3}{240} + \frac{4 \times 1 \times 3 \times 3}{240} + \frac{4 \times 3 \times 3 \times 1}{240} = \frac{27 + 36 + 36}{240}$$

$$= \frac{99}{240} = .413$$

34.

a. $\binom{20}{6} = 38{,}760.$ all from day shift) $= \dfrac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} = \dfrac{38{,}760}{8{,}145{,}060} = .0048$

b.

$$P(\text{all from same shift}) = \frac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} + \frac{\binom{15}{6}\binom{30}{0}}{\binom{45}{6}} + \frac{\binom{10}{6}\binom{35}{0}}{\binom{45}{6}}$$

$= .0048 + .0006 + .0000 = .0054$

c. P(at least two shifts represented) = 1 – P(all from same shift)
= 1 - .0054 = .9946

d. Let A_1 = day shift unrepresented, A_2 = swing shift unrepresented, and A_3 = graveyard shift unrepresented. Then we wish $P(A_1 \cup A_2 \cup A_3)$.

$P(A_1)$ = P(day unrepresented) = P(all from swing and graveyard)

$P(A_1) = \dfrac{\binom{25}{6}}{\binom{45}{6}}$, $P(A_2) = \dfrac{\binom{30}{6}}{\binom{45}{6}}$,

$P(A_3) = \dfrac{\binom{35}{6}}{\binom{45}{6}}$,

$P(A_1 \cap A_2)$ = P(all from graveyard) $= \dfrac{\binom{10}{6}}{\binom{45}{6}}$

$P(A_1 \cap A_3) = \dfrac{\binom{15}{6}}{\binom{45}{6}}$, $P(A_2 \cap A_3) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$, $P(A_1 \cap A_2 \cap A_3) = 0$,

$$\text{So } P(A_1 \cup A_2 \cup A_3) = \frac{\binom{25}{6}}{\binom{45}{6}} + \frac{\binom{30}{6}}{\binom{45}{6}} + \frac{\binom{35}{6}}{\binom{45}{6}} - \frac{\binom{10}{6}}{\binom{45}{6}} - \frac{\binom{15}{6}}{\binom{45}{6}} - \frac{\binom{20}{6}}{\binom{45}{6}}$$

$= .2939 - .0054 = .2885$

38.

f. P(selecting 2 - 75 watt bulbs) = $\dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{15 \cdot 9}{455} = .2967$

g. P(all three are the same) = $\dfrac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \dfrac{4+10+20}{455} = .0747$

h. $\binom{4}{1}\binom{5}{1}\binom{6}{1} = \dfrac{120}{455} = .2637$

i. To examine exactly one, a 75 watt bulb must be chosen first. (6 ways to accomplish this). To examine exactly two, we must choose another wattage first, then a 75 watt. (9×6 ways). Following the pattern, for exactly three, $9 \times 8 \times 6$ ways; for four, $9 \times 8 \times 7 \times 6$; for five, $9 \times 8 \times 7 \times 6 \times 6$.

P(examine at least 6 bulbs) = 1 – P(examine 5 or less)

= 1 – P(examine exactly 1 or 2 or 3 or 4 or 5)

= 1 – [P(one) + P(two) + … + P(five)]

$$= 1 - \left[\frac{6}{15} + \frac{9\times6}{15\times14} + \frac{9\times8\times6}{15\times14\times13} + \frac{9\times8\times7\times6}{15\times14\times13\times12} + \frac{9\times8\times7\times6\times6}{15\times14\times13\times12\times11}\right]$$

= 1 – [.4 + .2571 + .1582 + .0923 + .0503]

= 1 - .9579 = .0421

40.

j. If the A's are distinguishable from one another, and similarly for the B's, C's and D's, then there are 12! Possible chain molecules. Six of these are:

$A_1A_2A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_1A_3A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$

$A_2A_1A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_2A_3A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

$A_3A_1A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_3A_2A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

These 6 (=3!) differ only with respect to ordering of the 3 A's. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the A's is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (B's, C's and D's are still distinguishable). At this point there are $\frac{12!}{3!}$ molecules. Now suppressing subscripts on the B's, C's and D's in turn gives ultimately chain molecules.

k. Think of the group of 3 A's as a single entity, and similarly for the B's, C's, and D's. Then there are 4! Ways to order these entities, and thus 4! Molecules in which the A's are contiguous, the B's, C's, and D's are also. Thus, P(all together) = .

42.

Seats:

1	2	3	4	5	6

P(J&P in 1&2) $= \dfrac{2\times1\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1} = \dfrac{1}{15} = .0667$

P(J&P next to each other) = P(J&P in 1&2) + … + P(J&P in 5&6)

$$= 5\times\frac{1}{15} = \frac{1}{3} = .333$$

P(at least one H next to his W) = 1 – P(no H next to his W)

We count the # of ways of no H next to his W as follows:
if orderings without a H-W pair in seats #1 and 3 and no H next to his W = 6* × 4 × 1* × 2# × 1 × 1 = 48
*= pair, # =can't put the mate of seat #2 here or else a H-W pair would be in #5 and 6.

# of orderings without a H-W pair in seats #1 and 3, and no H next to his W = 6 × 4 × 2# × 2 × 2 × 1 = 192
#= can't be mate of person in seat #1 or #2.
So, # of seating arrangements with no H next to W = 48 + 192 = 240

And P(no H next to his W) = $= \dfrac{240}{6\times 5\times 4\times 3\times 2\times 1} = \dfrac{1}{3}$, so

P(at least one H next to his W) = 1 - $\dfrac{1}{3} = \dfrac{2}{3}$

48.

l. $P(A_2 \mid A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.06}{.12} = .50$

m. $P(A_1 \cap A_2 \cap A_3 \mid A_1) = \dfrac{.01}{.12} = .0833$

n. We want P[(exactly one) | (at least one)].
P(at least one) $= P(A_1 \cup A_2 \cup A_3)$
$= .12 + .07 + .05 - .06 - .03 - .02 + .01 = .14$
Also notice that the intersection of the two events is just the 1st event, since "exactly one" is totally contained in "at least one."
So P[(exactly one) | (at least one)]= $\dfrac{.04+.01}{.14} = .3571$

o. The pieces of this equation can be found in your answers to exercise 26 (section 2.2):
$$P(A_3' \mid A_1 \cap A_2) = \frac{P(A_1 \cap A_2 \cap A_3')}{P(A_1 \cap A_2)} = \frac{.05}{.06} = .833$$

50.

p. $P(M \cap LS \cap PR) = .05$, directly from the table of probabilities

q. $P(M \cap Pr) = P(M,Pr,LS) + P(M,Pr,SS) = .05+.07=.12$

r. P(SS) = sum of 9 probabilities in SS table = 56, P(LS) = 1 = .56 = .44

s. P(M) = .08+.07+.12+.10+.05+.07 = .49
P(Pr) = .02+.07+.07+.02+.05+.02 = .25

t. P(M|SS ∩ Pl) = $\dfrac{P(M \cap SS \cap Pl)}{P(SS \cap Pl)} = \dfrac{.08}{.04 + .08 + .03} = .533$

u. P(SS|M ∩ Pl) = $\dfrac{P(SS \cap M \cap Pl)}{P(M \cap Pl)} = \dfrac{.08}{.08 + .10} = .444$

P(LS|M Pl) = 1 - P(SS|M Pl) = 1 - .444 = .556