Matrix Inversion

I will just focus on inverting a 2×2 matrix. The usual procedure is to write the matrix next to the identity matrix.

$$\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array} \right).$$

We now use normal row operations to make the first matrix, the identity matrix. The matrix on the right will then be the inverse. We can start by adding $-\frac{c}{a}$ times the first row to the second.

$$\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{array} \right).$$

Now we take $-\frac{ab}{ad-cb}$ times the second row and add it to the first.

$$\left(\begin{array}{cc|cc} a & 0 & \frac{ad}{ad-bc} & -\frac{ab}{ad-bc} \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{array} \right).$$

We can now multiply the second row by $\frac{a}{ad-bc}$ and divide the first row by a to get,

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array} \right).$$

So we have,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

In class I had the matrix,

$$\begin{pmatrix} \frac{e^{-3t}+e^{-t}}{2} & \frac{-e^{-3t}+e^{-t}}{2} \\ \frac{-e^{-3t}+e^{-t}}{2} & \frac{e^{-3t}+e^{-t}}{2} \end{pmatrix}.$$

Plugging into the formula we developed gives the inverse,

$$\frac{1}{2e^{-4t}} \begin{pmatrix} e^{-3t}+e^{-t} & e^{-3t}-e^{-t} \\ e^{-3t}-e^{-t} & e^{-3t}+e^{-t} \end{pmatrix}.$$