

## Math 3120 – Differential Equations II

Homework #1 Due Friday January 22

1. Find the radius of convergence about the given point,  $x_0$ , for which the following differential equations with initial conditions given at  $x_0$  are guaranteed to have a unique solution analytic in the interval  $x_0 - R < x < x_0 + R$ :

(a)  $y'' + \frac{1}{1+2t}y' + \frac{t}{1-t^2}y = 0$ ,  $t_0 = 0$ .

(b)  $(1 - 9x^2)y'' + 4y' + xy = 0$ ,  $x_0 = 1$

2. Find the power series solution for the following about the given point. In each case determine the set of values of  $x$  for which the series converges and if possible sum the series in closed form.

(a)  $y' = xy$ ,  $y(0) = 5$  about the point  $x = 0$ .

(b)  $(1 + x)y'' + 2y' = 0$  about the point  $x = 1$

3. Find the first 5 terms of the power series of the solution to

$$y'' + e^x y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

For what values of  $x$  would you expect the series to converge.

4. Consider the following differential equation

$$(1 - x^2)y'' - xy' + \mu^2 y = 0.$$

- (a) Find and classify all singular points.
- (b) Find the recurrence relationship for a series expansion about  $x = 0$ .
- (c) Show that if  $\mu$  is an integer, the series solutions is just a polynomial of degree  $\mu$
- (d) The polynomial solutions to this equation are referred to as Chebyshev polynomials of the first kind. Find the first 3 such polynomials.
5. Find the general power series solution about  $x = 0$  for the differential equation

$$xy'' + 2y' + xy = 0.$$

6. Find the first 4 terms in the general power series solution to

$$y'' - xy' - y = 5\sqrt{x}$$

Hint: Find a particular solution of the form  $x^r(b_0 + b_1x + b_2x^2 + \dots)$  then solve  $y'' - xy' - y = 0$  as we have done previously.