

Math 3120 Practise Test

1. Find two independent power series solutions for,

$$y'' + xy = 0,$$

and discuss convergence (give the first 4 nonzero terms, you don't need to find a general formula for the n^{th} term).

2. For the differential equation,

$$2xy'' - (1+x)y' + 2y = 0,$$

- Show $x = 0$ is a regular singular point.
- Determine the indicial equation and its two roots.
- Write out the form of the series solution. If possible give the form for 2 independent solutions.
- Find the recurrence relation.

3. Given the following heat transfer problem:

$$\begin{aligned}u_t &= (0.5)u_{xx}, & 0 < x < 4, \\u(0, t) &= 0, \\u(4, t) &= 100, \\u(x, 0) &= 25x + 80 \sin(\pi x) \cos(\pi x).\end{aligned}$$

- Construct a linear function $v(x)$ such that $u(x, t) - v(x)$ is 0 at $x = 0$ and $x = 4$.
- Determine the equation the function $w(x, t) = u(x, t) - v(x)$ must satisfy.
- Use separation of variable and Fourier series to find a solution for w and then find u .

4. Consider the eigenvalue problem,

$$\begin{aligned}u'' + 2u' + u &= -\lambda u, \\u(0) &= 0, \\u(1) &= 0.\end{aligned}$$

- Recast the differential equation in the form $L(u) = (p(x)u')' - q(x)u = -\lambda r(x)u$. What are p , q and r ?
- Determine the eigenpairs.
- Verify that the eigenfunctions are orthogonal with respect to the appropriate dot product.

In addition to 4 questions like the above the test will also have a page of short answer type questions.

A formula sheet is allowed.