

Math League Team Questions

1. We say that a day is *astonishing* if the month, date, and year are all divisible by 3, when written in standard form (MM-DD-YY).

For example, yesterday (03 – 21 – 03) was an astonishing day because 03, 21, and 03 are all divisible by 3.

The first astonishing day in this decade occurred on March 3rd, 2003.

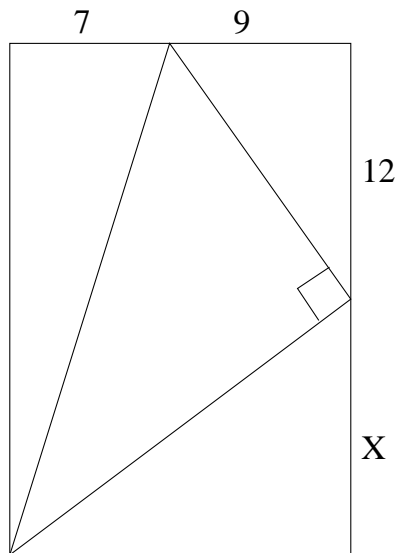
The last astonishing day in this decade will occur on December 30th, 2009.

Determine the total number of astonishing days in this decade.

2. Let a and b be real numbers. If $a - b = 2$ and $ab = 15$, determine *all* possible values of $a + b$.

(*Hint: there is more than one answer*).

3. In the figure, a right-angled triangle is inscribed in a rectangle.



Determine the value of x .

4. Let $S = \{3, 8, 13, 18, 23, 28, \dots, 1998, 2003\}$.

How many *perfect squares* appear in the set S ?

5. A hemispherical bowl is filled to half its depth with apple juice. Through what angle should the bowl be tilted so that the juice just starts to pour out?
6. Each day, Katie ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the third day, 128 remained. How many jellybeans were in the jar originally?
7. For this question, ABC will represent a three-digit integer with digits A , B , and C . So if $A = 3, B = 5, C = 7$, then $ABC = 357$, not $ABC = 3 \times 5 \times 7$.

Let ABC be a three-digit integer, all of whose digits are distinct and non-zero.

During a boring class, a student decides to write down the six permutations of the number, namely ABC , ACB , BAC , BCA , CAB , and CBA . She discovers that:

ABC is divisible by 2.

ACB is divisible by 3.

BAC is divisible by 4.

BCA is divisible by 5.

CAB is divisible by 6.

CBA is a divisor of 1995.

There is only one integer ABC satisfying all of the above conditions.

Find this three-digit integer ABC .

8. A parabola meets the y -axis at point A and the x -axis at points B and C . Let O be the origin.

There are two parabolas for which $OA = BC$ and the area of $\triangle ABC$ is 32.

Determine the equations of these two parabolas.

9. *This is a 10 minute question.*

Let a , b , and c represent the sides of an acute-angled triangle, and let x , y , and z represent the three altitudes of this triangle.

Find a triplet (a, b, c) such that *all six* of the numbers a, b, c, x, y, z are *distinct positive integers*.

(Note: there is more than one answer).