

Math League Game 4: Group Questions

You will solve the following ten problems together as a team. At the end of 40 minutes, we will collect your answer sheet. Your team will receive 3 points for each correct answer.

1. A *lattice point* is a point (x, y) , where the coordinates are both integers. For example, $(3, -4)$ and $(5, 0)$ are lattice points, but $(2, 4.58)$ is not.

Determine the number of lattice points on the circumference of the circle $x^2 + y^2 = 25$.

2. Let $S = \frac{2^2 - 1}{2^2} \times \frac{3^2 - 1}{3^2} \times \frac{4^2 - 1}{4^2} \times \dots \times \frac{2004^2 - 1}{2004^2}$.

Express S as a fraction, reduced to lowest terms.

3. A *palindrome* is a number that reads the same forwards and backwards, such as 8338 and 50705.

Let A and B be four-digit palindromes, and let C be a five-digit palindrome. If $A + B = C$, determine all possible values of C .

4. The circle with equation $x^2 + y^2 = 1$ intersects the line $y = 7x + 5$ at two distinct points A and B . Let O be the centre of the circle. Find the measure of $\angle AOB$.

5. Determine all integers x such that $(x^2 - 3x + 1)^{x+1} = 1$.

6. Let $f(a, b)$ denote the sum of the integers between a and b , inclusive. For example, $f(1, 5) = 1 + 2 + 3 + 4 + 5 = 15$ and $f(3, 6) = 3 + 4 + 5 + 6 = 18$.

Determine the value of $f(133333, 533333)$.

7. A hexagon and an equilateral triangle have equal perimeters. If the area of the hexagon is $6\sqrt{3}$ square units, what is the area of the triangle?

8. Determine all values of x for which

$$(1999x - 99)^3 = (1234x - 56)^3 + (765x - 43)^3.$$

9. Find all solutions (x, y) in real numbers to

$$\begin{aligned}\frac{1}{x} + \frac{1}{y} &= \frac{5}{6} \\ x^2y + xy^2 &= 30\end{aligned}$$

10. Find the number of real solutions to the equation $\sin(x) = \frac{x}{315}$.