Math League Game 4: Relay Questions

RELAY ONE: ten minutes

In this relay, the following context will be used for each of the four questions:

A wooden box with dimensions x cm by y cm by z cm has volume xyz cm^3. We say that such a box is an (x, y, z)-box. We will assume that each of x, y, z is at least 3.

All six faces of this (x, y, z)-box are painted red. The box is then cut into xyz equal unit cubes (i.e., cubes that measure 1 cm on each side). Each little cube must have either 0, 1, 2, or 3 of its faces painted red.

1. Let **A** be the number of cubes with no red faces in a $(3, 3, 3)$-box.

   Write the value of **A** in Box 1 of the Relay Answer Sheet.

2. In a $(3, 3, 3)$-box, there are **B** cubes that have exactly **A** of its faces painted red.

   Write the value of **B** in Box 2 of the Relay Answer Sheet.

3. Let **C** be the number of cubes with exactly *one* red face in a $(6, 6, \mathbf{B})$-box.

   Write the value of **C** in Box 3 of the Relay Answer Sheet.

4. Let **D** be the number of cubes with exactly *three* red faces in a $(\mathbf{C}, 2003, 2004)$-box.

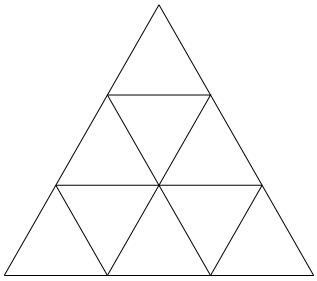
   Write the value of **D** in Box 4 of the Relay Answer Sheet.

   Hand in the completed sheet to your proctor.

RELAY TWO: ten minutes

In this relay, the following context will be used for each of the four questions:

Consider the following equilateral triangle of side length 3.



In this figure, there are exactly 13 triangles: nine triangles of side length 1, three triangles of side length 2, and one triangle of side length 3. Note that all the triangles are equilateral.

For an equilateral triangle of side length n, let $f(n)$ denote the number of triangles (of all sizes) that point *up*, and $g(n)$ denote the number of triangles (of all sizes) that point *down*.

For example, $f(3) = 10$ and $g(3) = 3$.

1. Let $\mathbf{A} = f(4)$.

   Write the value of $\mathbf{A}$ in Box 1 of the Relay Answer Sheet.

2. Let $\mathbf{B} = \mathbf{A} - g(4)$.

   Write the value of $\mathbf{B}$ in Box 2 of the Relay Answer Sheet.

3. $\mathbf{C}$ is the integer for which $g(\mathbf{C}) = \mathbf{B}$.

   Write the value of $\mathbf{C}$ in Box 3 of the Relay Answer Sheet.

4. Let $\mathbf{D}$ be the smallest integer such that $f(\mathbf{D} + 1) - f(\mathbf{D}) > \mathbf{C}^2$.

   Write the value of $\mathbf{D}$ in Box 4 of the Relay Answer Sheet.

   Hand in the completed sheet to your proctor.