

Sample Full Solution Problems for the Final Game

1. There are 5 red marbles and 6 green marbles in a jar. Zenia plays a strange game. She removes two marbles at a time, with the following rules:
 - (a) If the marbles are both green, she puts one green marble back.
 - (b) If there is one marble of each colour, she puts one red marble back.
 - (c) If the marbles are both red, she puts one green marble back.

Prove that no matter how Zenia selects the marbles, her final marble must always be red.

2. **Randy:** “Hi Rachel, that’s an interesting quadratic equation you have written down. What are its roots?”

Rachel: “The roots are two positive integers. One of the roots is my age, and the other root is the age of my younger brother, Jimmy.”

Randy: “That is very neat! Let me see if I can figure out how old you and Jimmy are. That shouldn’t be too difficult since all of your coefficients are integers. By the way, I notice that the sum of the three coefficients is a prime number.”

Rachel: “Interesting. Now figure out how old I am.”

Randy: “Instead, I will guess your age and substitute it for x in your quadratic equation . . . darn, that gives me -55 , and not 0 .”

Rachel: “Oh, leave me alone!”

- (a) Prove that Jimmy is two years old.
 - (b) Determine Rachel’s age.
3. Prima, Donna, and Karan participated in the “Math League Olympics”, in which there were M different “math-letic” events. In each event p points were awarded for first place, q points were awarded for second place, and r points were awarded for third place, where $p > q > r > 0$, and p, q, r are integers. Prima finished with 22 points, and Donna and Karan both finished with 9 points. Karan won the shot put. What is the value of M , and who finished second in the high jump?
 4. Let a, b, c, d, e be five positive integers satisfying the following conditions:
 - (i) $a < b < c < d < e$.
 - (ii) Exactly four of these five integers are prime.
 - (iii) $d = b + c$.
 - (iv) $a + b + c + d + e$ is a prime number.
 - (v) $\sqrt{e - a}$ is a prime number.

Determine the value of $a^3 + b^3 + c^3 + d^3 + e^3$.

5. Prove that $\sqrt{3}$ is an irrational number.
6. There are x books in the Old Testament, where x is a two-digit integer. If you multiply the digits of x you get the integer y , which is the number of books in the New Testament. Adding x and y , we get 66, the number of books in the Bible. Determine x and y .