

Power Questions

1. We rearrange the letters of the word “RESETS” to form new words like “RETSSE” (these don’t have to be English words). We put all these words into a list in alphabetic order, so the first word in the list is “EERSST” while the last one is “TSSREE”.
 - a) How many distinct words are in this list?
 - b) A word is selected at random from the list. What is the probability that both “S”s are next to each other?
 - c) What is the 139th word in this list?

2. Angela and Paul are playing a game with Angela playing first. They start with a positive integer n . On each player’s turn, they may move to a new integer by either dividing n by 2, or subtracting from n the largest power of 2 which is less than n . For example, if $n = 20$ then Angela could start by moving to 10 or 4. If a player cannot make a legal move, they lose.
 - a) Find all n for which Angela has two different choices for her first move.
 - b) Find all n with $1 \leq n \leq 10$ for which Angela will win the game.
 - c) Given a particular n , show that every sequence of play will have the same number of moves.

3. In the following triangle of numbers, each entry is the sum of the three numbers above it in the previous row. For instance, in the 5th row, the number 16 is determined by the sum of the numbers 3, 6 and 7 in the previous row.

				1					
			1	1	1				
		1	2	3	2	1			
	1	3	6	7	6	3	1		
1	4	10	16	19	16	10	4	1	
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

- a) Explain why the middle entry of every row is odd.
- b) Show that every row beyond the second contains at least one number which is even.