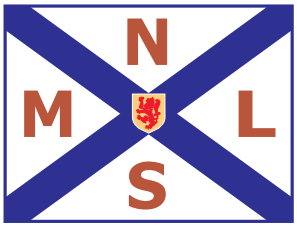

Nova Scotia
Math League

2007–2008

Provincial Final

Pairs Relay

A. Upon expansion, $(1+\sqrt{2})^4$ can be expressed in the form $a+b\sqrt{2}$ for a, b positive integers. Let $\texttt{A} = a - b$.

Pass on `A`.

B. You will receive `A`.

At *Cafe Tesfa Sahaiye*, one chai latte and one muffin costs \$4, while one chai latte and six muffins costs \$20. Let `B` be the cost of `A` chai lattes.

Pass on `B`.

C. You will receive `B`.

Let `C` be the number of positive integers of length `B` consisting only of digits 1 and 2 which are divisible by 4.

Pass on `C`.

D. You will receive `C`.

Let `D` be the sum

$$(\texttt{C}+1)+(\texttt{C}-2)+(\texttt{C}+3)+(\texttt{C}-4)+\ldots+(\texttt{C}+99)+(\texttt{C}-100)$$

Pass on `D`

Full solutions

A. Expanding

$$(1+\sqrt{2})^2 = 1+2\sqrt{2}+2 = 3+2\sqrt{2}$$
$$(3+2\sqrt{2})^2 = 9+12\sqrt{2}+8 = 17+12\sqrt{2}$$

Thus $a = 17$ and $b = 12$, so $\texttt{A} = a - b = 17 - 12 = 5$.

B. Let c be the cost of a chai latte and m the cost of a muffin. Then we have

$$c + m = 4 \tag{1}$$
$$c + 6m = 20 \tag{2}$$

Then 6 (1) - (2) gives $5c = 4$ or $c = \frac{4}{5}$. Thus the cost of $\texttt{A}$ chai lattes is $\texttt{A}\frac{4}{5}$. Since $\texttt{A} = 5$, have $\texttt{B} = 4$.

C. A number is divisible by 4 if the number formed by the last two digits is divisible by 4. Using only 1's and 2's, the only such two digit number is 12. It does not matter what the remaining $\texttt{B} - 2$ digits are, so $\texttt{C} = 2^{\texttt{B}-2}$. Since $\texttt{B} = 4$, have $\texttt{C} = 4$.

If the rule for divisibility by four is unknown, then writing out the sixteen possibilities once $\texttt{B}$ is obtained and checking is a more tedious method of obtaining the result.

D. There are 100 terms in the series, so there are 100 copies of $\texttt{C}$. Therefore $\texttt{D} = 100 \cdot \texttt{C} + x$ for some value x. It remains to determine the value of x.

Pair up the non-$\texttt{C}$ terms as follows:

$$1 - 2 = -1$$
$$3 - 4 = -1$$
$$5 - 6 = -1$$
$$\vdots$$
$$99 - 100 = -1$$

Then all non-$\texttt{C}$ terms are accounted for and their sum is -50. Therefore $\texttt{D} = 100 \cdot \texttt{C} - 50$. Since $\texttt{C} = 4$, have $\texttt{D} = 350$.