Nova Scotia Math League

2007–2008

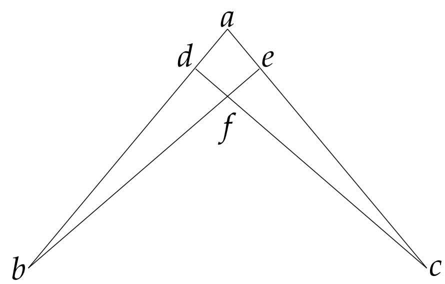
Game Two

Individual Relay

A. Let `A` be the sum of all prime numbers between 90 and 100.

Pass on `A`.

B. You will receive `A`.



In the above diagram:

- $\angle bdf = 90°$,
- $\angle cef = 90°$,
- $\angle bac =$ `A`°, and
- $\angle bfc =$ `B`°.

Pass on `B`.

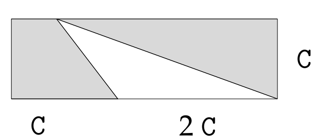
C. You will receive `B`.

An urn contains `B` red balls and 4 blue balls. After adding `C` red balls and 1 blue ball to the urn, the ratio of red balls to blue balls is 120:1.

Pass on `C`.

D. You will receive `C`.

Let `D` be the area of the shaded region in the diagram below:



Pass on `D`

Solutions

A. The numbers in question are 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, and 100. We can eliminate all even numbers, along with 95 (multiple of 5), 93 and 99 (clearly multiples of 3). The numbers which remain are 91 and 97. Some inspection shows that 91 = 7 $\times$13, while 97 is prime. Hence the desired sum is simply 97.

   Note: See if you can find a trick for determining whether a number is divisible by seven without just trying to divide it by 7.

B. We have the following information:

$$\begin{aligned} \angle abf &= 90 - \mathtt{A} && \text{(angles in a triangle sum to 180)} \\ \angle bdf &= \angle adf = 90 && \text{(angles in a straight line sum to 180)} \end{aligned}$$

   Then in $\triangle bdf$, we have $\angle bfd = \mathtt{A}$. Therefore $\angle bfc = 180 - \mathtt{A}$. Since $\mathtt{A}$ = 97, we have $\mathtt{B}$ = $\angle bfc = 83$.

   Alternative Solution: We have $\angle efd = \angle bfc = \mathtt{B}$ (opposite angles). But the angles of quadrilateral $\square adfe$ sum to 360, so

$$\begin{aligned} 360 &= \angle bac + \angle aef + \angle efd + \angle fda \\ &= \mathtt{A} + 90 + \mathtt{B} + 90. \end{aligned}$$

   Therefore $\mathtt{B} = 180 - \mathtt{A} = 180 - 97 = 83$.

C. We know that $\frac{\mathtt{B}+\mathtt{C}}{4+1} = \frac{120}{1}$. Solving for $\mathtt{C}$ gives $\mathtt{C} = 600 - \mathtt{B}$. Since $\mathtt{B} = 83$, we get $\mathtt{C}$ = 517.

D. The unshaded area forms a triangle with height $\mathtt{C}$ and base 2 $\mathtt{C}$, hence area $\mathtt{C}^2$. The area of the entire shape is $\mathtt{C} \cdot (\mathtt{C} + 2\mathtt{C}) = 3\mathtt{C}^2$. Therefore the area of the shaded region is the difference, namely $2\mathtt{C}^2$. Since $\mathtt{C}$ = 517, we obtain $\mathtt{D}$ = 534 578.