Nova Scotia Math League

2007–2008

Game Three

INDIVIDUAL RELAY

A. Let `A` be the smallest, positive number such that

- when divided by 2, the remainder is 0;
- when divided by 3, the remainder is 2;
- when divided by 5, the remainder is 0.

Pass on `A`.

B. You will receive `A`.

The average (arithmetic mean) of a set of fifty numbers is `A`. Two numbers, 23 and 17, are deleted from the set. Let `B` be the average of the new set of numbers.

Pass on `B`.

C. You will receive `B`.

The product of the roots of the quadratic

$$p(x) = x^2 - 9x + \mathtt{B}$$

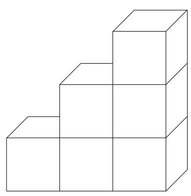
equals the product of the roots of the quadratic

$$q(x) = \left(x + \frac{10}{3}\right)(x + \mathtt{C})$$

Pass on `C`.

D. You will receive `C`.

Six identical cubes make up the solid drawn below. The solid has volume `C`. Let `D` be the surface area of the solid.



Pass on `D`

Solutions

A. Since when divided by 2 and 5, the remainder is 0, we are looking for multiples of 10. The remainder of 10 divided by 3 is 1. The remainder of 20 divided by 3 is 2. Therefore 20 is our answer.

B. Since the average of the set of fifty numbers is $\mathtt{A}$, then the sum of all fifty numbers is $50\mathtt{A}$. Removing 17 and 23 from the set, the sum of the new set of numbers is $50\mathtt{A} - 17 - 23 = 50\mathtt{A} - 40$. There are now 48 numbers in the set, so $\mathtt{B} = \dfrac{50\mathtt{A} - 40}{48}$. Since $\mathtt{A} = 20$, we have $\mathtt{B} = 20$.

C. The product of the roots of $p(x)$ is $\mathtt{B}$ since if $p(x)$ has roots a and b, then

$$p(x) = (x - a)(x - b) = x^2 + (-a - b)x + ab$$

and equating coefficients gives that $ab = \mathtt{B}$. The product of the roots of $q(x)$ is $\mathtt{C}\left(\frac{10}{3}\right)$ since the roots are $-\mathtt{C}$ and $-\frac{10}{3}$. Therefore

$$\mathtt{B} = \mathtt{C}\left(\frac{10}{3}\right)$$
$$\mathtt{C} = \frac{3}{10}\mathtt{B}.$$

Since $\mathtt{B} = 20$, we have $\mathtt{C} = 6$.

D. Let x be the area of one of the faces of any of the cubes. There are 24 exposed faces, so the surface area of the solid is $24x$. Thus $\mathtt{D} = 24x$. It remains to determine x.

Since the volume of the solid is $\mathtt{C}$ and each cube is of equal volume and there are six cubes, the volume of an individual cube is $\dfrac{\mathtt{C}}{6}$. Therefore, the area of one of the faces of any cube is $\left(\dfrac{\mathtt{C}}{6}\right)^{\frac{1}{3}}$. From above, we then get that $\mathtt{D} = 24\left(\dfrac{\mathtt{C}}{6}\right)^{\frac{1}{3}}$. Since $\mathtt{C} = 6$, we get that $\mathtt{D} = 24$.