

Nova Scotia

Math League

2007–2008

Game Three

PAIRS RELAY

A. Let A be such that the distance between $(0, A)$ and $(-2, 0)$ equals the distance between $(0, A)$ and $(4, 6)$.

Pass on A.

B. You will receive A.

$$\text{Let } f(x) = 3x - 1 \text{ and } g(x) = x^2.$$

$$\text{Let } B = -f(g(A)) + g(f(-A)).$$

Pass on B.

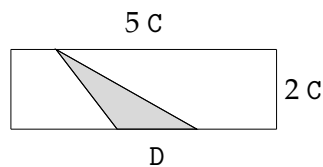
C. You will receive B.

A worm crawled forward a total of B centimetres over four consecutive days. Each day she crawled 12 more centimetres than the day before. Let C be the number of centimetres she crawled on the third day.

Pass on C.

D. You will receive C.

A rectangle has sides of length $2C$ and $5C$. A triangle with base of length D is drawn inside the rectangle. The area of the triangle is $\frac{1}{5}$ the area of the rectangle.



Pass on D

Full solutions

A. The distance between point $(0, A)$ and $(-2, 0)$ is

$$\sqrt{(0 - (-2))^2 + (A - 0)^2} = \sqrt{4 + A^2}.$$

The distance between point $(0, A)$ and $(4, 6)$ is

$$\sqrt{(0 - 4)^2 + (A - 6)^2} = \sqrt{A^2 - 12A + 52}$$

Since these distances are equal, we have

$$\sqrt{4 + A^2} = \sqrt{A^2 - 12A + 52}$$

$$4 + A^2 = A^2 - 12A + 52$$

$$12A = 48$$

$$A = 4.$$

B. We have

$$g(A) = A^2 \implies f(g(A)) = 3(A^2) - 1 = 3A^2 - 1,$$

and

$$f(-A) = -3A - 1 \implies g(f(-A)) = (-3A - 1)^2 = 9A^2 + 6A + 1.$$

Therefore

$$\begin{aligned} B &= -f(g(A)) + g(f(-A)) \\ &= -3A^2 + 1 + 9A^2 + 6A + 1 \\ &= 6A^2 + 6A + 2 \end{aligned}$$

Since $A = 4$, this gives $B = 6(4)^2 + 6(4) + 2 = 122$.

C. Let C be the amount she crawled on the third day. Then on the four days, she crawled the following distances: $C - 24$, $C - 12$, C , and $C + 12$. Since she crawled a total of B centimetres, we have

$$B = C - 24 + C - 12 + C + C + 12$$

$$B = 4C - 24$$

$$C = \frac{B}{4} + 6$$

Since $B = 122$, we have $C = 36.5$.

D. The area of the rectangle is $10C^2$. The area of the triangle is CD . Since the area of the triangle is $\frac{1}{5}$ the area of the rectangle, we have

$$\frac{1}{5}10C^2 = CD$$

$$2C = D$$

Since $C = 36.5$, we have $D = 73$.