

September 2008 Questions

1) Let $S(n)$ be the sum of the first n terms of the following sequence:

$$0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, \dots$$

(a) Show that

$$S(n) = \begin{cases} \frac{n^2}{4} & \text{if } n \text{ is even;} \\ \frac{n^2 - 1}{4} & \text{if } n \text{ is odd.} \end{cases}$$

(b) Show $S(a + b) - S(a - b) = ab$ where a, b are positive integers with $a > b$.

2) An equilateral triangle of area $3\sqrt{3}$ is inscribed in a circle. What is the radius of the circle?

Submit all solutions by 23.59 September 30, 2008.