Nova Scotia Math League

2010–2011

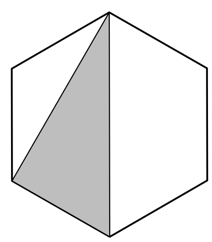
Game Three

PROBLEMS

Team Questions

1) Billy has \$7.00 in dimes, nickels, and quarters. The total value of his dimes is twice the value of his quarters and one-half the value of his nickels. How many coins does Billy have?

2) The area of the regular hexagon shown below is 12. Find the area of the shaded triangle.



3) Consider points $P = (0, 4)$ and $Q = (7, 25)$ in the (x, y)-plane. Find the coordinates of the point R on the parabola $y = x^2$ such that $|PR| + |QR|$ is as small as possible.

(As usual, $|AB|$ denotes the length of the line segment between A and B.)

4) A street busker draws you into the following game: He first shows you three cards, one green, one red, and one blue. The cards are placed in separate but identical envelopes, which the busker then shuffles behind his back. Finally, he presents you with the envelopes and asks you to guess which card is in each envelope. You win the game if ***all*** of your guesses are ***incorrect***.

What is the probability that you win the game?

5) Bob buys a number of widgets for a total cost of \$840. Later he discovers that another store is selling the same widgets for \$7 less per unit. Bob calculates that if he had spent his \$840 at the cheaper store, he could have bought an additional 4 widgets. How many widgets did Bob purchase?

6) Consider the circles defined by the equations

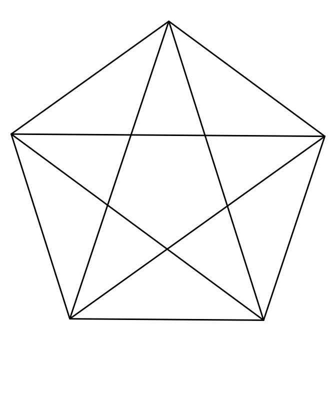
$$(x+3)^2 + (y-2)^2 = 1$$

and

$$(x-5)^2 + (y-5)^2 = 4.$$

How many circles are tangent to *both* these circles and also pass through the origin $(0,0)$?

7) How many triangles can be found in the figure below?



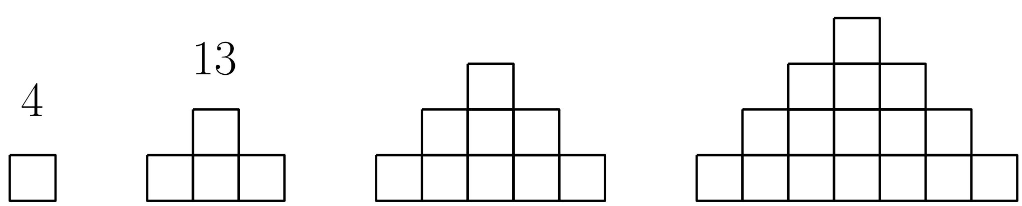
8) The product $25! \cdot 24! \cdot 23! \cdot \cdots \cdot 3! \cdot 2! \cdot 1!$ evaluates to a very large integer that ends in a number of zeros. How many?

(Recall that $n!$ represents the product $n(n-1)(n-2) \cdot 3 \cdot 2 \cdot 1$.)

9) At "Johnny's Take-Out", chicken nuggets come in boxes of 4 or 9. What is the largest number of chicken nuggets that one ***cannot*** order exactly?

(For example: One can order 17 nuggets, since $17 = 2 \cdot 4 + 9$, but one cannot order exactly 11 nuggets.)

10) A child uses matchsticks to construct the "pyramid" shapes shown below.



Each edge of each small square is a matchstick, so the first pyramid (a square) is made from 4 matchsticks, the second pyramid (with two levels) requires 13, etc.

How many matchsticks are needed to make a pyramid with 100 levels?

Pairs Relay

A. The sequence $a_0, a_1, a_2, \ldots$ is defined as follows:

- $a_0 = 1$
- $a_n = \dfrac{1}{1 + a_{n-1}}$ for $n \geq 1$.

Let A be the denominator of a_8 (expressed as a fraction in lowest terms).

Pass on A

B. You will receive A.

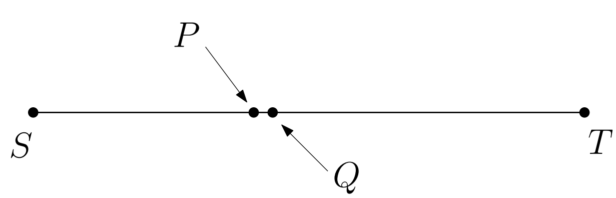
At a party there are a total of A girls. Each girl is either blonde or brunette, and has either blue or brown eyes. You observe that there are 33 brunettes, 35 brown-eyed girls, and 12 blue-eyed blondes.

Let B be the number of brown-eyed brunettes at the party.

Pass on B

C. You will receive B.

Line segment ST is B units long. The points P and Q lie on ST as shown below, with P dividing ST in the ratio 2:3 and Q dividing ST in the ratio 3:4.

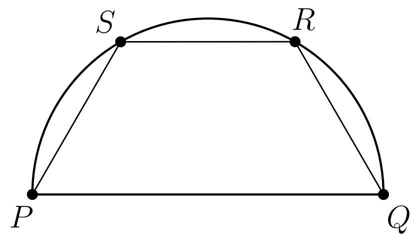


Let C be the length of segment PQ.

Pass on C

D. You will receive `C`.

Quadrilateral $PQRS$ is inscribed in a semicircle whose diameter PQ is of length 14. Side RS is parallel to PQ and is `C` times its length.



Let `D` be the area of quadrilateral $PQRS$.

Done!

Individual Relay

A. The sum of the squares of the three sides of a right-angled triangle is 128. Let A be the length of the hypotenuse of the triangle.

Pass on A

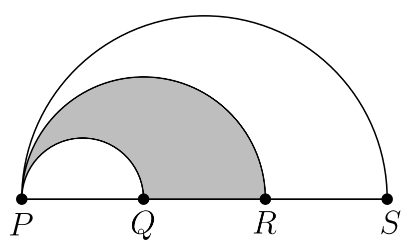
B. You will receive A.

You randomly take a handful of coins from a jar containing pennies and dimes. Let B be the minimum number of coins you must remove to guarantee you will have at least 11 pennies or at least A dimes in your selection.

Pass on B

C. You will receive B.

Three semicircles are nested as shown in the diagram. Segments PQ, QR, and RS are all of the same length, and the large semicircle has area B.



Let C be the area of the shaded region.

Pass on C

D. You will receive C.

Let D be the sum of all real numbers x that satisfy $|x + \mathtt{C}| = 2|x - \mathtt{C}|$.

Done!