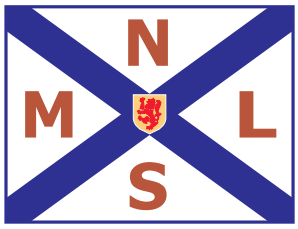


Nova Scotia
Math League

2014–2015

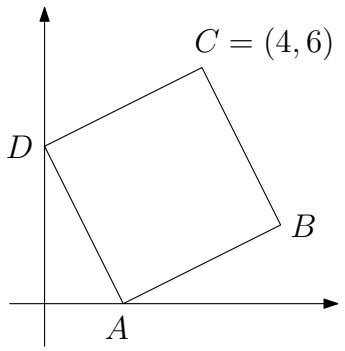
Game Two

PROBLEMS

Team Questions

1. In how many distinct ways can you make change for \$10 using only quarters and dimes?

2. Square $ABCD$ has vertices A and D on the x- and y-axes, respectively, and $C = (4, 6)$. Find the area of this square.

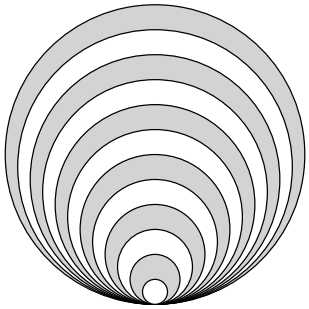


3. Find the units digit of

$$1 + 3 + 3^2 + 3^3 + 3^4 + \cdots + 3^{2015}.$$

4. An isosceles triangle has sides of lengths 18 and 41. Compute the area of the triangle.

5. In the diagram below, the twelve circles have diameters 1 through 12 and are mutually tangent. Find area of the shaded region.



6. Bob and Jim go for a run on a 400m circular track. They start at the same place and at the same time. Bob runs clockwise around the track at 13.4 km/h and Jim runs counterclockwise at 10.2 km/h. What is the straight-line distance between them after 15 minutes?

7. A coin is weighted so that heads are more likely than tails. When it is flipped twice there is a 48% chance of obtaining one head and one tail. What is the probability of heads on a single flip of the coin?

8. Let $f(x) = x^2 - 10x + 30$. Find all x such that $f(f(x)) = f(x)$.

9. The weather last June was very unpleasant. Every day was cloudy, rainy, or windy. On average, 5 out of 6 days were cloudy and 4 out of 5 were rainy or windy. It was never rainy without being cloudy, but 1/3 of the windy days weren't cloudy. It was rainy the same number of days as it was windy.

   On how many days was it both rainy and windy?

   Note: There are 30 days in June!

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10. Each province and territory of Canada is to be coloured either red, green, or yellow in such a way that any two provinces/territories that share a border must have distinct colours. In how many ways can this be done?

Note:

- Regions that meet only at a "corner" are not considered to share a border. For instance, Saskatchewan is not adjacent to Nunavut.
- All land under the jurisdiction of any given province/territory must have the same colour. For instance, Newfoundland and Labrador must be similarly coloured, as do Baffin Island and Nunavut.

Pairs Relay

P-A. Compute

$$\texttt{A} = \frac{(4^2+4^2+4^2)(6^3+6^3+6^3)}{(2^4+2^4+2^4+2^4)(3^3+3^3+3^3+3^3+3^3+3^3)}.$$

Pass on A

P-B. You will receive A.

Let r and s be the roots of $x^2 - \texttt{A}x + 1$, where $r < s$.

Let $\texttt{B} = (r+1)(s+1)$.

Pass on B

P-C. You will receive B.

Let C be the area of the triangle bounded between the graphs of $y = |x - \texttt{B}|$ and $y = \texttt{B}$.

Pass on C

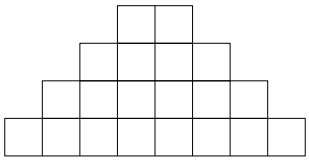
P-D. You will receive C.

The sum of the **squares** of two consecutive **odd** positive integers is $\texttt{C}(\texttt{C}+1)$. Let D be the smaller of these integers.

Done!

Individual Relay

I-A. A pyramid is formed by stacking unit squares as shown below. Let A be the perimeter of the pyramid that has 48 squares on its bottom row.



Pass on A

I-B. You will receive A.

Solve for B:

$$\frac{1}{\frac{1}{\sqrt{\mathtt{A}}}+\frac{1}{\mathtt{B}}}=\frac{\sqrt{\mathtt{A}}}{3}.$$

Pass on B

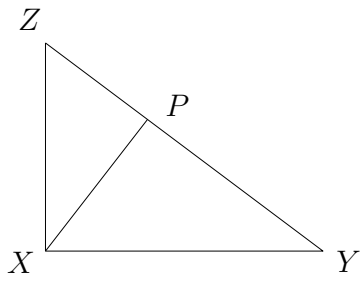
I-C. You will receive B.

Today, Doug is twice as old as John. In B years, Doug will be twice as old as James. Let C be the age difference (in years) between John and James.

Pass on C

I-D. You will receive C.

Right triangle XYZ has legs $|XZ| = \mathtt{C}$ and $|XY| = \mathtt{C}+1$. Point P is on hypotenuse YZ such that $XP \perp YZ$. Find $|XP|$.



Done!