

Nova Scotia

Math League

2015–2016

Game One

PROBLEMS

Team Questions

1. All of Bill's math tests are graded out of 100 points. If Bill scores 95% on his next test, then his average for the year increases by 3 points. If he scores 75%, then his average decreases by 1 point.

How many math tests has Bill written so far this year?

2. Jane and Tracy begin running around a 300m oval track. They start from the same spot but head out in opposite directions, with Jane running at 6 m/s and Tracy jogging at half that speed.

How far (in metres) has Jane run when she meets Tracy for the second time?

Note: Their starting point is not considered their first meeting!

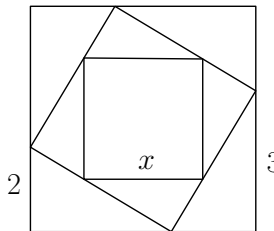
3. Find n such that

$$\frac{1 + 3 + 5 + 7 + \cdots + (2n - 1)}{2 + 4 + 6 + 8 + \cdots + 2n} = \frac{2015}{2016}.$$

4. Let $N = \overbrace{11111 \cdots 1111}^{1000 \text{ ones}}$ and let $M = 2015N$.

Calculate the sum of the digits of M .

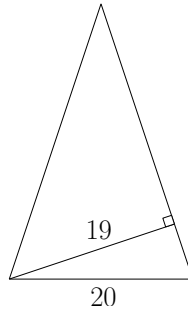
5. The figure below shows three squares nested within each other, with the sides of the smallest and largest squares parallel. Find x , the length of each side of the smallest square.



6. The function f satisfies $f(x) = 3f(1 - x) + x^3$ for all real numbers x .

Determine $f(3)$.

7. An isosceles triangle has base 20 and the altitude to one of its equal sides is 19. Find the length of the two equal sides.

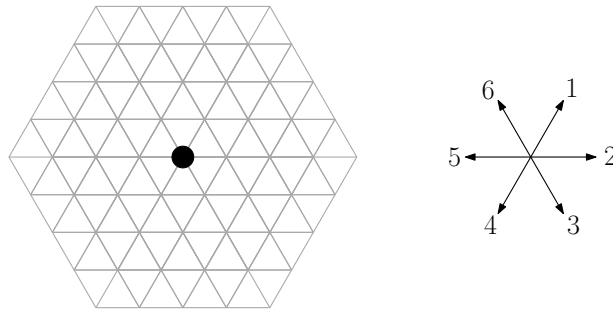


8. Determine the sum of the cubes of the three roots of $x^3 - 2x + 1 = 0$.
9. A magician lays 5 playing cards on a table. He asks you to select any number of the cards (possibly none at all). He removes your selected cards from the table and then asks you to flip over any number of those remaining.

In how many ways can you perform the magician's tasks?

Note: The order in which you flip over the cards does not matter.

10. A bead is placed at a vertex of a hexagonal lattice. It is pushed around the lattice, traversing one edge at a time, with the direction of each move randomly determined by the throw of a 6-sided die (see diagram).



Find the probability that the bead returns to its starting position after 4 moves.

Pairs Relay

P-A. The faces of a strange 6-sided die are numbered 1, 1, 2, 3, 5, 8. The die is rolled twice and the results are summed.

Let A be the smallest integer greater than 1 that is **not** a possible sum.

Pass on A

P-B. You will receive A.

The difference and the quotient of two numbers both equal A.

Let B be the **smaller** number.

Pass on B

P-C. You will receive B. Suppose $B = p/q$, expressed in lowest terms.

The lines $y = 3x - p$ and $y = -x + q$ intersect at a point P . Let (m, n) be the point nearest to P that has integer coordinates.

Let $C = mn$.

Pass on C

P-D. You will receive C.

Donuts cost \$1 each, but for every 3 you purchase you get one free. Suppose you leave the store with C donuts.

Let D be the number of dollars you spent.

Done!

Individual Relay

I-A. My two sons share the same birthday, but they were born three years apart. Their ages are now $\frac{1}{3}$ and $\frac{1}{4}$ of mine.

Let A be my age.

Pass on A

I-B. You will receive A .

If I received \sqrt{A} more points on my test, my grade would increase from 76% to 84%.

Let B be the total number of points on the test.

Pass on B

I-C. You will receive B .

Let C be the smallest possible value of $(p - B)^2$, where p is a prime number.

Pass on C

I-D. You will receive C .

Let D be the maximum value that can be obtained by inserting a single pair of parentheses (brackets) into the expression

$$10 + 3 \times C + 14$$

Done!

Team Questions Answer Key

1. 4
2. 400
3. 2015
4. 8000
5. $\frac{13}{5}$
6. $-\frac{3}{8}$
7. $\frac{200}{\sqrt{39}}$
8. -3
9. 243 ($= 3^5$)
10. $\frac{5}{72}$

Pairs Relay Answer Key

- A. 12
- B. $\frac{12}{11}$
- C. 30
- D. 23

Individual Relay Answer Key

- A. 36
- B. 75
- C. 4
- D. 66