Nova Scotia
Math League

2017–2018

Game One

PROBLEMS

Team Questions

1. A large vat contains 80L of 6% salt solution. It is desired to raise the salt concentration to 7% by adding some quantity of 10% salt solution. How many litres of 10% solution should be added?

2. Find the largest integer n such that n^2 divides 10!.

   (Recall that $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.)

3. In the figure below, the triangle is equilateral with side length 2, and one of its sides serves as the diameter of the semicircle. Find the area of the shaded region.
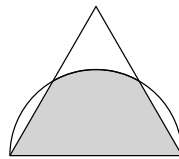

4. How many times per day are the minute and hour hands on a clock perpendicular? (Assume the hands move smoothly, rather than jumping precisely on the minute or hour marks.)

5. Bobby stacks unit cubes to make a tower like the one shown below. He continues to expand the tower until it is 10 units tall. Find the surface area (not including base) of this tower.
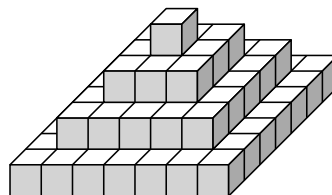
6. A rectangle is dissected into several smaller rectangles, all congruent, as shown below. The area of the large rectangle is 88 square units. Find its perimeter.
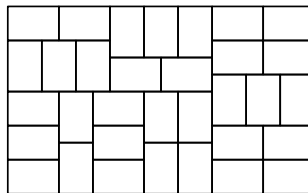
7. Six points are equispaced around a circle. Three of the points are chosen at random and joined by lines to form a triangle. Find the probability that this triangle is isosceles.
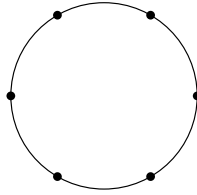
8. The hypotenuse of a right triangle is 6 and its perimeter is 14. Find the area of the triangle.

9. Find the sum of all fractions p/q between 0 and 1 that have denominator 100 when expressed in lowest terms.

10. How many quadratic functions $p(x) = ax^2 + bx + c$ satisfy

$$p(x-1)p(x)p(x+1) = 0 \qquad \text{for } x = 1,2,3,4,5$$

and $p(0) = 1$?

Pairs Relay

P-A. A group of A colleagues decide to buy an expensive coffee maker for the office, splitting the cost evenly amongst themselves. If 3 more people joined the coffee club, the per person cost would be 20% less.

<div align="right">| Pass on A |</div>

P-B. You will receive A.

It rained on A days of John's vacation, but never in both the morning and afternoon of the same day. The morning was clear on 10 days and the afternoon was clear on 12.

Let B be the length (in days) of John's vacation. | Pass on B |

P-C. You will receive B.

Let S be the sum of the B smallest positive multiples of B.

Let $C = \sqrt{S}$. | Pass on C |

P-D. You will receive C.

Bob's average grade on his first 4 tests was 59% and his average on the next 3 was C%. There were 10 tests altogether and his overall average was 56%.

Let D be his percent average on his final 3 tests. | Done! |

Individual Relay

I-A. Let A be the number of integers between 1 and 40 inclusive that are divisible by either 2 or 3 (or both).

I-B. You will receive A.

The sum of two numbers is $\frac{5}{3}$A and their difference is A. Let B be the square root of their product.

I-C. You will receive B.

For nonzero real numbers x, y, define $x \boxplus y$ by the rule

$$x \boxplus y := \frac{x}{y} + \frac{y}{x}.$$

Let $C = (3 \boxplus B) \boxplus (B \boxplus 3)$.

I-D. You will receive C.

Suppose $x : y = (C + 1) : (C + 2)$. Let $D = \dfrac{2x + 3y}{2x - 3y}$.

Team Questions Answer Key

1. $\dfrac{80}{3}$

2. 720

3. $\dfrac{\pi}{6} + \dfrac{\sqrt{3}}{2}$

4. 44

5. 761

6. $\dfrac{116}{3}$

7. $\dfrac{2}{5}$

8. 7

9. 20

10. 3

Pairs Relay Answer Key

A. 12

B. 17

C. 51

D. 57

Individual Relay Answer Key

A. 27

B. 18

C. 2

D. -3