Nova Scotia Math League

2017–2018

**Game Two**

PROBLEMS

Team Questions

1. Emily numbers the pages of her math course notes, starting as usual with $1, 2, 3, \ldots$.. In doing so, she writes exactly 999 digits.

   How many pages of notes does Emily have?

2. Amy and Sarah run a race. Amy runs 25% faster than Sarah and finishes the race 15 seconds ahead. How long does it take Amy to finish the race?

3. Today is both my and my son's birthday. I am now twice as old as my father was on the day I was born. Coincidentally, on the day my son was born, my father was twice my age! My father is now 78 years old. How old is my son?

4. Find the area of the dashed rectangle.
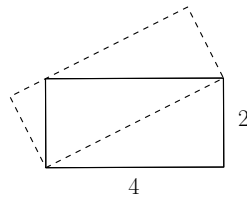
5. I recently bought some bitcoin. Over the first week, I lost X% of my investment. But bitcoin then gained $2X$% in the following week, bringing my two week profit to 8%.

   Find all possible values of X.

6. Find the number of digits in the decimal expansion of 20^{30}.

7. You begin with a 1L jug of wine. One cup (250mL) of wine is removed and replaced with water, and the container is thoroughly mixed. This process is repeated twice more. What is the final ratio of water to wine in the jug?

8. The faces of a $4 \times 4 \times 4$ cube are painted red. The cube is then sliced into 64 unit cubes. One of these cubes is selected at random and thrown like a die. Determine the probability that the face showing is painted.

9. A running track is composed of three adjacent lanes, each 1m wide, with parallel straightaways and semicircular curves. The distance around the **inside** of the inner-most lane is precisely 400m. Find the area of the track.
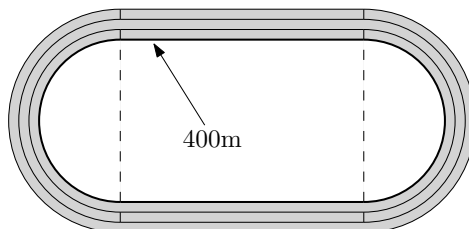
10. Two perpendicular lines pass through the point $(3, 7)$. The distance between their x-intercepts is 15. Find the distance between their y-intercepts.

Pairs Relay

P-A. Increasing the length of each side of a cube by 1 cm increases its surface area by 66 cm^2.

Let A be the corresponding increase in volume (measured in cm^3).

$\boxed{\text{Pass on A}}$

P-B. You will receive A.

The average of two numbers is A/7. If one of the numbers is doubled and the other is tripled then their average becomes 30.

Let B be the smaller of the numbers. $\boxed{\text{Pass on B}}$

P-C. You will receive B.

A closed rectangular box is twice as long as it is wide, and twice as tall as it is long. Its volume is B times its surface area.

Let C be the length of the shortest side of the box. $\boxed{\text{Pass on C}}$
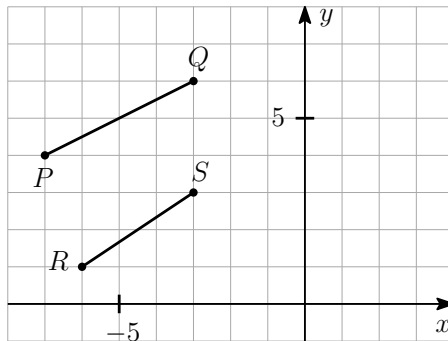
P-D. You will receive C.

A ship sails 34 km north, followed by C km east, then C km south, and finally 36 km west.

Let D be the distance between the ship and its starting point.

$\boxed{\text{Done!}}$

Individual Relay

I-A. If segments $\overline{PQ}$ and $\overline{RS}$ are extended, the lines intersect at the point (x, y). Let A $= x$.

<div align="right">

Pass on A

</div>

I-B. You will receive A.

Find B such that
$$\frac{81^B}{27^A} = \frac{1}{3}.$$

<div align="right">

Pass on B

</div>

I-C. You will receive B.

Let C be the sum of the first B terms of the series
$$1 - 4 + 7 - 10 + 13 - 16 + \cdots$$

<div align="right">

Pass on C

</div>

I-D. You will receive C.

Compute
$$D = \frac{(1 + \sqrt{C})^4 - (1 - \sqrt{C})^4}{1 + C}.$$

<div align="right">

Done!

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Team Questions Answer Key

1. 369

2. 60 seconds

3. 26 years old

4. 8

5. 10 or 40

6. 40

7. $\dfrac{27}{37}$

8. $\dfrac{1}{4}$

9. $1200 + 9\pi$ square metres

10. $\dfrac{45}{7}$

Pairs Relay Answer Key

A. 91

B. 8

C. 28

D. 10

Individual Relay Answer Key

A. 15

B. 11

C. 16

D. 32