6. $\frac{d}{dx}\left(2\sqrt{x}+\sqrt{y}\right)=\frac{d}{dx}(3) \;\Rightarrow\; 2\cdot\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\cdot y'=0 \;\Rightarrow\; \frac{1}{\sqrt{x}}+\frac{y'}{2\sqrt{y}}=0 \;\Rightarrow$

$\frac{y'}{2\sqrt{y}}=-\frac{1}{\sqrt{x}} \;\Rightarrow\; y'=-\frac{2\sqrt{y}}{\sqrt{x}}$

10. $\frac{d}{dx}(y^5+x^2y^3)=\frac{d}{dx}\left(1+ye^{x^2}\right) \;\Rightarrow\; 5y^4\,y'+(x^2\cdot 3y^2\,y'+y^3\cdot 2x)=0+y\cdot e^{x^2}\cdot 2x+e^{x^2}\cdot y' \;\Rightarrow$

$y'\left(5y^4+3x^2y^2-e^{x^2}\right)=2xye^{x^2}-2xy^3 \;\Rightarrow\; y'=\frac{2xy\left(e^{x^2}-y^2\right)}{5y^4+3x^2y^2-e^{x^2}}$

12. $\frac{d}{dx}(1+x)=\frac{d}{dx}\left[\sin(xy^2)\right] \;\Rightarrow\; 1=[\cos(xy^2)](x\cdot 2y\,y'+y^2\cdot 1) \;\Rightarrow\; 1=2xy\cos(xy^2)\,y'+y^2\cos(xy^2) \;\Rightarrow$

$1-y^2\cos(xy^2)=2xy\cos(xy^2)\,y' \;\Rightarrow\; y'=\frac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}$

22. $\frac{d}{dx}[g(x)+x\sin g(x)]=\frac{d}{dx}(x^2) \;\Rightarrow\; g'(x)+x\cos g(x)\cdot g'(x)+\sin g(x)\cdot 1=2x.$ If $x=0$, we have

$g'(0)+0+\sin g(0)=2(0) \;\Rightarrow\; g'(0)+\sin 0=0 \;\Rightarrow\; g'(0)+0=0 \;\Rightarrow\; g'(0)=0.$

36. $x^4+y^4=a^4 \;\Rightarrow\; 4x^3+4y^3\,y'=0 \;\Rightarrow\; 4y^3\,y'=-4x^3 \;\Rightarrow\; y'=-x^3/y^3 \;\Rightarrow$

$y''=-\left(\frac{y^3\cdot 3x^2-x^3\cdot 3y^2\,y'}{(y^3)^2}\right)=-3x^2y^2\cdot\frac{y-x(-x^3/y^3)}{y^6}=-3x^2\cdot\frac{y^4+x^4}{y^4y^3}=-3x^2\cdot\frac{a^4}{y^7}=\frac{-3a^4x^2}{y^7}$

46. $y=\sqrt{\tan^{-1}x}=(\tan^{-1}x)^{1/2} \;\Rightarrow$

$y'=\frac{1}{2}(\tan^{-1}x)^{-1/2}\cdot\frac{d}{dx}(\tan^{-1}x)=\frac{1}{2\sqrt{\tan^{-1}x}}\cdot\frac{1}{1+x^2}=\frac{1}{2\sqrt{\tan^{-1}x}\,(1+x^2)}$

48. $g(x)=\sqrt{x^2-1}\,\sec^{-1}x \;\Rightarrow\; g'(x)=\sqrt{x^2-1}\cdot\frac{1}{x\sqrt{x^2-1}}+\sec^{-1}x\cdot\frac{1}{2}\left(x^2-1\right)^{-1/2}(2x)=\frac{1}{x}+\frac{x\sec^{-1}x}{\sqrt{x^2-1}}$

$\left[\text{or } \frac{\sqrt{x^2-1}+x^2\sec^{-1}x}{x\sqrt{x^2-1}}\right]$

4. $f(x)=\ln(\sin^2 x)=\ln(\sin x)^2=2\ln|\sin x| \;\Rightarrow\; f'(x)=2\cdot\frac{1}{\sin x}\cdot\cos x=2\cot x$

6. $f(x)=\log_5(xe^x) \;\Rightarrow\; f'(x)=\frac{1}{xe^x\ln 5}\frac{d}{dx}(xe^x)=\frac{1}{xe^x\ln 5}(xe^x+e^x\cdot 1)=\frac{e^x(x+1)}{xe^x\ln 5}=\frac{x+1}{x\ln 5}$

Another solution: We can change the form of the function by first using logarithm properties.

$f(x)=\log_5(xe^x)=\log_5 x+\log_5 e^x \;\Rightarrow\; f'(x)=\frac{1}{x\ln 5}+\frac{1}{e^x\ln 5}\cdot e^x=\frac{1}{x\ln 5}+\frac{1}{\ln 5}$ or $\frac{1+x}{x\ln 5}$

16. $y=\frac{1}{\ln x}=(\ln x)^{-1} \;\Rightarrow\; y'=-1(\ln x)^{-2}\cdot\frac{1}{x}=\frac{-1}{x(\ln x)^2}$

34. $y = \ln(x^3 - 7) \;\Rightarrow\; y' = \dfrac{1}{x^3 - 7} \cdot 3x^2 \;\Rightarrow\; y'(2) = \dfrac{12}{8-7} = 12$, so an equation of a tangent line at $(2, 0)$ is

$y - 0 = 12(x - 2)$ or $y = 12x - 24$.

38. $y = \sqrt{x}\, e^{x^2} (x^2 + 1)^{10} \;\Rightarrow\; \ln y = \ln\sqrt{x} + \ln e^{x^2} + \ln(x^2+1)^{10} \;\Rightarrow\; \ln y = \frac{1}{2}\ln x + x^2 + 10\ln(x^2+1) \;\Rightarrow$

$\dfrac{1}{y} y' = \dfrac{1}{2} \cdot \dfrac{1}{x} + 2x + 10 \cdot \dfrac{1}{x^2+1} \cdot 2x \;\Rightarrow\; y' = \sqrt{x}\, e^{x^2} (x^2+1)^{10} \left(\dfrac{1}{2x} + 2x + \dfrac{20x}{x^2+1} \right)$

48. $y = (\ln x)^{\cos x} \;\Rightarrow\; \ln y = \cos x \ln(\ln x) \;\Rightarrow\; \dfrac{y'}{y} = \cos x \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x} + (\ln\ln x)(-\sin x) \;\Rightarrow$

$y' = (\ln x)^{\cos x} \left(\dfrac{\cos x}{x \ln x} - \sin x \ln\ln x \right)$

10. (a) At maximum height the velocity of the ball is 0 ft/s. $v(t) = s'(t) = 80 - 32t = 0 \;\Leftrightarrow\; 32t = 80 \;\Leftrightarrow\; t = \frac{5}{2}$.

So the maximum height is $s\left(\frac{5}{2}\right) = 80\left(\frac{5}{2}\right) - 16\left(\frac{5}{2}\right)^2 = 200 - 100 = 100$ ft.

(b) $s(t) = 80t - 16t^2 = 96 \;\Leftrightarrow\; 16t^2 - 80t + 96 = 0 \;\Leftrightarrow\; 16(t^2 - 5t + 6) = 0 \;\Leftrightarrow\; 16(t-3)(t-2) = 0$.

So the ball has a height of 96 ft on the way up at $t = 2$ and on the way down at $t = 3$. At these times the velocities are $v(2) = 80 - 32(2) = 16$ ft/s and $v(3) = 80 - 32(3) = -16$ ft/s, respectively.

18. $V(t) = 5000\left(1 - \frac{1}{40}t\right)^2 \;\Rightarrow\; V'(t) = 5000 \cdot 2\left(1 - \frac{1}{40}t\right)\left(-\frac{1}{40}\right) = -250\left(1 - \frac{1}{40}t\right)$

(a) $V'(5) = -250\left(1 - \frac{5}{40}\right) = -218.75$ gal/min

(b) $V'(10) = -250\left(1 - \frac{10}{40}\right) = -187.5$ gal/min

(c) $V'(20) = -250\left(1 - \frac{20}{40}\right) = -125$ gal/min

(d) $V'(40) = -250\left(1 - \frac{40}{40}\right) = 0$ gal/min

The water is flowing out the fastest at the beginning—when $t = 0$, $V'(t) = -250$ gal/min. The water is flowing out the slowest at the end—when $t = 40$, $V'(t) = 0$. As the tank empties, the water flows out more slowly.

4. (a) $y(t) = y(0)e^{kt} \;\Rightarrow\; y(2) = y(0)e^{2k} = 600$, $y(8) = y(0)e^{8k} = 75{,}000$. Dividing these equations, we get

$e^{8k}/e^{2k} = 75{,}000/600 \;\Rightarrow\; e^{6k} = 125 \;\Rightarrow\; 6k = \ln 125 = \ln 5^3 = 3\ln 5 \;\Rightarrow\; k = \frac{3}{6}\ln 5 = \frac{1}{2}\ln 5$.

Thus, $y(0) = 600/e^{2k} = 600/e^{\ln 5} = \frac{600}{5} = 120$.

(b) $y(t) = y(0)e^{kt} = 120e^{(\ln 5)t/2}$ or $y = 120 \cdot 5^{t/2}$

(c) $y(5) = 120 \cdot 5^{5/2} = 120 \cdot 25\sqrt{5} = 3000\sqrt{5} \approx 6708$ bacteria.

(d) $y(t) = 120 \cdot 5^{t/2} \;\Rightarrow\; y'(t) = 120 \cdot 5^{t/2} \cdot \ln 5 \cdot \frac{1}{2} = 60 \cdot \ln 5 \cdot 5^{t/2}$.

$y'(5) = 60 \cdot \ln 5 \cdot 5^{5/2} = 60 \cdot \ln 5 \cdot 25\sqrt{5} \approx 5398$ bacteria/hour.

(e) $y(t) = 200{,}000 \;\Leftrightarrow\; 120e^{(\ln 5)t/2} = 200{,}000 \;\Leftrightarrow\; e^{(\ln 5)t/2} = \frac{5000}{3} \;\Leftrightarrow\; (\ln 5)t/2 = \ln\frac{5000}{3} \;\Leftrightarrow$

$t = \left(2\ln\frac{5000}{3}\right)/\ln 5 \approx 9.2$ h.

12. From the information given, we know that $\frac{dy}{dx} = 2y \quad \Rightarrow \quad y = Ce^{2x}$ by Theorem 2. To calculate C we use the point $(0, 5)$: $5 = Ce^{2(0)} \quad \Rightarrow \quad C = 5$. Thus, the equation of the curve is $y = 5e^{2x}$.

16. $\frac{dT}{dt} = k(T - 20)$. Let $y = T - 20$. Then $\frac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$. $\quad y(0) = T(0) - 20 = 95 - 20 = 75$, so $y(t) = 75e^{kt}$. When $T(t) = 70$, $\frac{dT}{dt} = -1°\text{C/min}$. Equivalently, $\frac{dy}{dt} = -1$ when $y(t) = 50$. Thus, $-1 = \frac{dy}{dt} = ky(t) = 50k$ and $50 = y(t) = 75e^{kt}$. The first relation implies $k = -1/50$, so the second relation says $50 = 75e^{-t/50}$. Thus, $e^{-t/50} = \frac{2}{3} \quad \Rightarrow \quad -t/50 = \ln\left(\frac{2}{3}\right) \quad \Rightarrow \quad t = -50 \ln\left(\frac{2}{3}\right) \approx 20.27$ min.