2. Quotient Rule: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \dfrac{x - 3x^{3/2}}{x^{1/2}} \quad \Rightarrow$

$$F'(x) = \frac{x^{1/2}\left(1 - \frac{9}{2}x^{1/2}\right) - \left(x - 3x^{3/2}\right)\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2})^2} = \frac{x^{1/2} - \frac{9}{2}x - \frac{1}{2}x^{1/2} + \frac{3}{2}x}{x} = \frac{\frac{1}{2}x^{1/2} - 3x}{x} = \frac{1}{2}x^{-1/2} - 3$$

Simplifying first: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \sqrt{x} - 3x = x^{1/2} - 3x \quad \Rightarrow \quad F'(x) = \frac{1}{2}x^{-1/2} - 3$ (equivalent).

For this problem, simplifying first seems to be the better method.

4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \quad \Rightarrow \quad g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

8. $f(t) = \dfrac{2t}{4+t^2} \quad \overset{\text{QR}}{\Rightarrow} \quad f'(t) = \dfrac{(4+t^2)(2) - (2t)(2t)}{(4+t^2)^2} = \dfrac{8 + 2t^2 - 4t^2}{(4+t^2)^2} = \dfrac{8 - 2t^2}{(4+t^2)^2}$

18. $y = \dfrac{1}{s + ke^s} \quad \overset{\text{QR}}{\Rightarrow} \quad y' = \dfrac{(s+ke^s)(0) - (1)(1+ke^s)}{(s+ke^s)^2} = -\dfrac{1+ke^s}{(s+ke^s)^2}$

26. $f(x) = \dfrac{ax+b}{cx+d} \quad \Rightarrow \quad f'(x) = \dfrac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} = \dfrac{acx + ad - acx - bc}{(cx+d)^2} = \dfrac{ad - bc}{(cx+d)^2}$

28. $f(x) = x^{5/2}e^x \quad \Rightarrow \quad f'(x) = x^{5/2}e^x + e^x \cdot \frac{5}{2}x^{3/2} = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x \quad \left[\text{or } \frac{1}{2}x^{3/2}e^x(2x+5)\right] \quad \Rightarrow$

$$f''(x) = \left(x^{5/2} + \tfrac{5}{2}x^{3/2}\right)e^x + e^x\left(\tfrac{5}{2}x^{3/2} + \tfrac{15}{4}x^{1/2}\right) = \left(x^{5/2} + 5x^{3/2} + \tfrac{15}{4}x^{1/2}\right)e^x \quad \left[\text{or } \tfrac{1}{4}x^{1/2}e^x(4x^2 + 20x + 15)\right]$$

30. $f(x) = \dfrac{x}{3+e^x} \quad \Rightarrow \quad f'(x) = \dfrac{(3+e^x)(1) - x(e^x)}{(3+e^x)^2} = \dfrac{3 + e^x - xe^x}{(3+e^x)^2} \quad \Rightarrow$

$$f''(x) = \frac{(3+e^x)^2\left[e^x - (xe^x + e^x \cdot 1)\right] - (3 + e^x - xe^x)(9 + 6e^x + e^x \cdot e^x)'}{\left[(3+e^x)^2\right]^2}$$

$$= \frac{(3+e^x)^2(-xe^x) - (3 + e^x - xe^x)(6e^x + e^x \cdot e^x + e^x \cdot e^x)}{(3+e^x)^4}$$

$$= \frac{(3+e^x)^2(-xe^x) - (3 + e^x - xe^x)(6e^x + 2e^{2x})}{(3+e^x)^4} = \frac{(3+e^x)^2(-xe^x) - (3 + e^x - xe^x)(2e^x)(3+e^x)}{(3+e^x)^4}$$

$$= \frac{(3+e^x)e^x\left[(3+e^x)(-x) - 2(3 + e^x - xe^x)\right]}{(3+e^x)^4} = \frac{e^x(-3x - xe^x - 6 - 2e^x + 2xe^x)}{(3+e^x)^3}$$

$$= \frac{e^x(xe^x - 2e^x - 3x - 6)}{(3+e^x)^3}$$

32. $y = \dfrac{e^x}{x} \quad \Rightarrow \quad y' = \dfrac{x \cdot e^x - e^x \cdot 1}{x^2} = \dfrac{e^x(x-1)}{x^2}$.

At $(1, e)$, $y' = 0$, and an equation of the tangent line is $y - e = 0(x - 1)$, or $y = e$.

4. $y = 2\csc x + 5\cos x \quad \Rightarrow \quad y' = -2\csc x\,\cot x - 5\sin x$

12. $y = \dfrac{1-\sec x}{\tan x} \quad \Rightarrow$

$$y' = \frac{\tan x\,(-\sec x\,\tan x) - (1-\sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x\,(-\tan^2 x - \sec x + \sec^2 x)}{\tan^2 x} = \frac{\sec x\,(1-\sec x)}{\tan^2 x}$$

16. Using Exercise 3.2.55(a), $f(x) = x^2 \sin x\,\tan x \quad \Rightarrow$

$$f'(x) = (x^2)' \sin x\,\tan x + x^2(\sin x)' \tan x + x^2 \sin x\,(\tan x)' = 2x\sin x\,\tan x + x^2\cos x\,\tan x + x^2\sin x\,\sec^2 x$$
$$= 2x\sin x\,\tan x + x^2\sin x + x^2 \sin x\,\sec^2 x = x\sin x\,(2\tan x + x + x\sec^2 x).$$

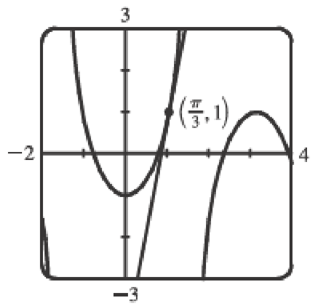
26. (a) $y = \sec x - 2\cos x \quad \Rightarrow \quad y' = \sec x\,\tan x + 2\sin x \quad \Rightarrow$

the slope of the tangent line at $\left(\frac{\pi}{3}, 1\right)$ is

$\sec\frac{\pi}{3}\tan\frac{\pi}{3} + 2\sin\frac{\pi}{3} = 2\cdot\sqrt{3} + 2\cdot\frac{\sqrt{3}}{2} = 3\sqrt{3}$, and an equation

is $y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$, or $y = 3\sqrt{3}\,x + 1 - \pi\sqrt{3}$.

(b)



30. $f(x) = \sec x \quad \Rightarrow \quad f'(x) = \sec x\,\tan x \quad \Rightarrow \quad f''(x) = \sec x\,(\sec^2 x) + \tan x\,(\sec x\,\tan x) = \sec x\,(\sec^2 x + \tan^2 x)$.

$f''\left(\frac{\pi}{4}\right) = \sqrt{2}\left[\left(\sqrt{2}\right)^2 + 1^2\right] = \sqrt{2}\,(2+1) = 3\sqrt{2}$

34. $y = \dfrac{\cos x}{2+\sin x} \quad \Rightarrow \quad y' = \dfrac{(2+\sin x)(-\sin x) - \cos x\,\cos x}{(2+\sin x)^2} = \dfrac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2} = \dfrac{-2\sin x - 1}{(2+\sin x)^2} = 0$ when

$-2\sin x - 1 = 0 \quad \Leftrightarrow \quad \sin x = -\frac{1}{2} \quad \Leftrightarrow \quad x = \frac{11\pi}{6} + 2\pi n$ or $x = \frac{7\pi}{6} + 2\pi n$, n an integer. So $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$ and

the points on the curve with horizontal tangents are: $\left(\frac{11\pi}{6} + 2\pi n, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}}\right)$, n an integer.

44. $\displaystyle\lim_{t\to 0}\frac{\sin^2 3t}{t^2} = \lim_{t\to 0}\left(\frac{\sin 3t}{t}\cdot\frac{\sin 3t}{t}\right) = \lim_{t\to 0}\frac{\sin 3t}{t}\cdot\lim_{t\to 0}\frac{\sin 3t}{t} = \left(\lim_{t\to 0}\frac{\sin 3t}{t}\right)^2 = \left(3\lim_{t\to 0}\frac{\sin 3t}{3t}\right)^2 = (3\cdot 1)^2 = 9$

2. Let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u} = u^{1/2}$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \dfrac{3}{2\sqrt{u}} = \dfrac{3}{2\sqrt{4+3x}}$.

4. Let $u = g(x) = \sin x$ and $y = f(u) = \tan u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\sec^2 u)(\cos x) = \sec^2(\sin x)\cdot\cos x$,

or equivalently, $[\sec(\sin x)]^2\cos x$.

6. Let $u = g(x) = e^x$ and $y = f(u) = \sin u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\cos u)(e^x) = e^x\cos e^x$.

10. $f(x)=(1+x^4)^{2/3} \quad\Rightarrow\quad f'(x)=\frac{2}{3}(1+x^4)^{-1/3}(4x^3)=\dfrac{8x^3}{3\sqrt[3]{1+x^4}}$

14. $y=a^3+\cos^3 x \quad\Rightarrow\quad y'=3(\cos x)^2(-\sin x)$ $\quad$ [a^3 is just a constant] $\quad =-3\sin x\cos^2 x$

32. $y=\tan^2(3\theta)=(\tan 3\theta)^2 \quad\Rightarrow\quad y'=2(\tan 3\theta)\cdot\dfrac{d}{d\theta}(\tan 3\theta)=2\tan 3\theta\cdot\sec^2 3\theta\cdot 3=6\tan 3\theta\sec^2 3\theta$

34. $y=x\sin\dfrac{1}{x} \quad\Rightarrow\quad y'=\sin\dfrac{1}{x}+x\cos\dfrac{1}{x}\left(-\dfrac{1}{x^2}\right)=\sin\dfrac{1}{x}-\dfrac{1}{x}\cos\dfrac{1}{x}$

38. $y=e^{k\tan\sqrt{x}} \quad\Rightarrow\quad y'=e^{k\tan\sqrt{x}}\cdot\dfrac{d}{dx}\left(k\tan\sqrt{x}\right)=e^{k\tan\sqrt{x}}\left(k\sec^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2}\right)=\dfrac{k\sec^2\sqrt{x}}{2\sqrt{x}}\,e^{k\tan\sqrt{x}}$