

Assignment 1 Solutions

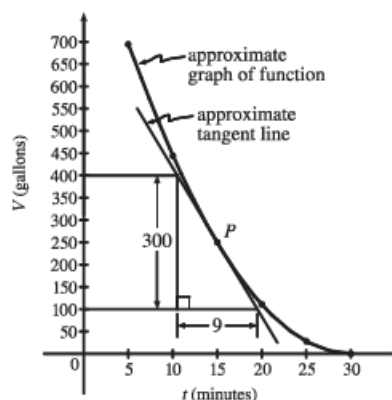
1. (a) Using $P(15, 250)$, we construct the following table:

t	Q	slope = m_{PQ}
5	(5, 694)	$\frac{694-250}{5-15} = -\frac{444}{10} = -44.4$
10	(10, 444)	$\frac{444-250}{10-15} = -\frac{194}{5} = -38.8$
20	(20, 111)	$\frac{111-250}{20-15} = -\frac{139}{5} = -27.8$
25	(25, 28)	$\frac{28-250}{25-15} = -\frac{222}{10} = -22.2$
30	(30, 0)	$\frac{0-250}{30-15} = -\frac{250}{15} = -16.\bar{6}$

(b) Using the values of t that correspond to the points closest to P ($t = 10$ and $t = 20$), we have

$$\frac{-38.8 + (-27.8)}{2} = -33.3$$

(c) From the graph, we can estimate the slope of the tangent line at P to be $\frac{-300}{9} = -33.\bar{3}$.



6. (a) $y = y(t) = 10t - 1.86t^2$. At $t = 1$, $y = 10(1) - 1.86(1)^2 = 8.14$. The average velocity between times 1 and $1 + h$ is

$$v_{\text{ave}} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{[10(1+h) - 1.86(1+h)^2] - 8.14}{h} = \frac{6.28h - 1.86h^2}{h} = 6.28 - 1.86h, \text{ if } h \neq 0.$$

(i) $[1, 2]: h = 1, v_{\text{ave}} = 4.42 \text{ m/s}$

(ii) $[1, 1.5]: h = 0.5, v_{\text{ave}} = 5.35 \text{ m/s}$

(iii) $[1, 1.1]: h = 0.1, v_{\text{ave}} = 6.094 \text{ m/s}$

(iv) $[1, 1.01]: h = 0.01, v_{\text{ave}} = 6.2614 \text{ m/s}$

(v) $[1, 1.001]: h = 0.001, v_{\text{ave}} = 6.27814 \text{ m/s}$

(b) The instantaneous velocity when $t = 1$ (h approaches 0) is 6.28 m/s.

4. (a) $\lim_{x \rightarrow 0} f(x) = 3$

(b) $\lim_{x \rightarrow 3^-} f(x) = 4$

(c) $\lim_{x \rightarrow 3^+} f(x) = 2$

(d) $\lim_{x \rightarrow 3} f(x)$ does not exist because the limits in part (b) and part (c) are not equal.

(e) $f(3) = 3$

8. (a) $\lim_{x \rightarrow 2} R(x) = -\infty$

(b) $\lim_{x \rightarrow 5} R(x) = \infty$

(c) $\lim_{x \rightarrow -3^-} R(x) = -\infty$

(d) $\lim_{x \rightarrow -3^+} R(x) = \infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = 2$, and $x = 5$.

20. For $f(x) = x \ln(x + x^2)$:

x	$f(x)$
1	0.693147
0.5	-0.143841
0.1	-0.220727
0.05	-0.147347
0.01	-0.045952
0.005	-0.026467
0.001	-0.006907

It appears that $\lim_{x \rightarrow 0^+} x \ln(x + x^2) = 0$.

28. $\lim_{x \rightarrow 5^-} \frac{e^x}{(x-5)^3} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \rightarrow 5^-$.

30. $\lim_{x \rightarrow \pi^-} \cot x = \lim_{x \rightarrow \pi^-} \frac{\cos x}{\sin x} = -\infty$ since the numerator is negative and the denominator approaches 0 through positive values as $x \rightarrow \pi^-$.