14. $\lim\limits_{x\to\infty}\sqrt{\dfrac{12x^3-5x+2}{1+4x^2+3x^3}} = \sqrt{\lim\limits_{x\to\infty}\dfrac{12x^3-5x+2}{1+4x^2+3x^3}}$ [Limit Law 11]

$= \sqrt{\lim\limits_{x\to\infty}\dfrac{12-5/x^2+2/x^3}{1/x^3+4/x+3}}$ [divide by x^3]

$= \sqrt{\dfrac{\lim\limits_{x\to\infty}(12-5/x^2+2/x^3)}{\lim\limits_{x\to\infty}(1/x^3+4/x+3)}}$ [Limit Law 5]

$= \sqrt{\dfrac{\lim\limits_{x\to\infty}12-\lim\limits_{x\to\infty}(5/x^2)+\lim\limits_{x\to\infty}(2/x^3)}{\lim\limits_{x\to\infty}(1/x^3)+\lim\limits_{x\to\infty}(4/x)+\lim\limits_{x\to\infty}3}}$ [Limit Laws 1 and 2]

$= \sqrt{\dfrac{12-5\lim\limits_{x\to\infty}(1/x^2)+2\lim\limits_{x\to\infty}(1/x^3)}{\lim\limits_{x\to\infty}(1/x^3)+4\lim\limits_{x\to\infty}(1/x)+3}}$ [Limit Laws 7 and 3]

$= \sqrt{\dfrac{12-5(0)+2(0)}{0+4(0)+3}}$ [Theorem 5 of Section 2.5]

$= \sqrt{\dfrac{12}{3}} = \sqrt{4} = 2$

26. $\lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right) = \lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)\left[\dfrac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}\right] = \lim\limits_{x\to-\infty}\dfrac{x^2-(x^2+2x)}{x-\sqrt{x^2+2x}}$

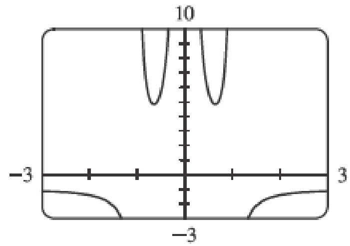
$= \lim\limits_{x\to-\infty}\dfrac{-2x}{x-\sqrt{x^2+2x}} = \lim\limits_{x\to-\infty}\dfrac{-2}{1+\sqrt{1+2/x}} = \dfrac{-2}{1+\sqrt{1+2(0)}} = -1$

Note: In dividing numerator and denominator by x, we used the fact that for $x<0$, $x=-\sqrt{x^2}$.

42. $\lim\limits_{x\to\infty}\dfrac{1+x^4}{x^2-x^4} = \lim\limits_{x\to\infty}\dfrac{\dfrac{1+x^4}{x^4}}{\dfrac{x^2-x^4}{x^4}} = \lim\limits_{x\to\infty}\dfrac{\dfrac{1}{x^4}+1}{\dfrac{1}{x^2}-1} = \dfrac{\lim\limits_{x\to\infty}\left(\dfrac{1}{x^4}+1\right)}{\lim\limits_{x\to\infty}\left(\dfrac{1}{x^2}-1\right)} = \dfrac{\lim\limits_{x\to\infty}\dfrac{1}{x^4}+\lim\limits_{x\to\infty}1}{\lim\limits_{x\to\infty}\dfrac{1}{x^2}-\lim\limits_{x\to\infty}1}$

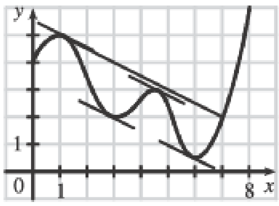
$= \dfrac{0+1}{0-1} = -1,$ so $y=-1$ is a horizontal asymptote.

$y=f(x)=\dfrac{1+x^4}{x^2-x^4}=\dfrac{1+x^4}{x^2(1-x^2)}=\dfrac{1+x^4}{x^2(1+x)(1-x)}$. The denominator is zero when $x=0,-1$, and 1, but the numerator is nonzero, so $x=0$, $x=-1$, and $x=1$ are vertical asymptotes. Notice that as $x\to 0$, the numerator and denominator are both positive, so $\lim\limits_{x\to 0}f(x)=\infty$. The graph confirms our work.



4. $f(x) = \cos 2x$, $[\pi/8, 7\pi/8]$. f, being the composite of the cosine function and the polynomial $2x$, is continuous and differentiable on $\mathbb{R}$, so it is continuous on $[\pi/8, 7\pi/8]$ and differentiable on $(\pi/8, 7\pi/8)$. Also, $f\left(\frac{\pi}{8}\right) = \frac{1}{2}\sqrt{2} = f\left(\frac{7\pi}{8}\right)$.
$f'(c) = 0 \quad \Leftrightarrow \quad -2\sin 2c = 0 \quad \Leftrightarrow \quad \sin 2c = 0 \quad \Leftrightarrow \quad 2c = \pi n \quad \Leftrightarrow \quad c = \frac{\pi}{2}n$. If $n = 1$, then $c = \frac{\pi}{2}$, which is in the open interval $(\pi/8, 7\pi/8)$, so $c = \frac{\pi}{2}$ satisfies the conclusion of Rolle's Theorem.

8. $\dfrac{f(7) - f(1)}{7 - 1} = \dfrac{2 - 5}{6} = -\dfrac{1}{2}$. The values of c which satisfy $f'(c) = -\frac{1}{2}$ seem to be about $c = 1.1, 2.8, 4.6$, and 5.8.



14. $f(x) = \dfrac{x}{x + 2}$, $[1, 4]$. f is continuous on $[1, 4]$ and differentiable on $(1, 4)$. $f'(c) = \dfrac{f(b) - f(a)}{b - a} \quad \Leftrightarrow$
$\dfrac{2}{(c+2)^2} = \dfrac{\frac{2}{3} - \frac{1}{3}}{4 - 1} \quad \Leftrightarrow \quad (c + 2)^2 = 18 \quad \Leftrightarrow \quad c = -2 \pm 3\sqrt{2}$. $-2 + 3\sqrt{2} \approx 2.24$ is in $(1, 4)$.

24. If $3 \le f'(x) \le 5$ for all x, then by the Mean Value Theorem, $f(8) - f(2) = f'(c) \cdot (8 - 2)$ for some c in $[2, 8]$. (f is differentiable for all x, so, in particular, f is differentiable on $(2, 8)$ and continuous on $[2, 8]$. Thus, the hypotheses of the Mean Value Theorem are satisfied.) Since $f(8) - f(2) = 6f'(c)$ and $3 \le f'(c) \le 5$, it follows that
$6 \cdot 3 \le 6f'(c) \le 6 \cdot 5 \quad \Rightarrow \quad 18 \le f(8) - f(2) \le 30$.

10. (a) $f(x) = 4x^3 + 3x^2 - 6x + 1 \quad \Rightarrow \quad f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x - 1)(x + 1)$. Thus, $f'(x) > 0 \quad \Leftrightarrow \quad x < -1$ or $x > \frac{1}{2}$ and $f'(x) < 0 \quad \Leftrightarrow \quad -1 < x < \frac{1}{2}$. So f is increasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$ and f is decreasing on $\left(-1, \frac{1}{2}\right)$.

(b) f changes from increasing to decreasing at $x = -1$ and from decreasing to increasing at $x = \frac{1}{2}$. Thus, $f(-1) = 6$ is a local maximum value and $f\left(\frac{1}{2}\right) = -\frac{3}{4}$ is a local minimum value.

(c) $f''(x) = 24x + 6 = 6(4x + 1)$. $f''(x) > 0 \quad \Leftrightarrow \quad x > -\frac{1}{4}$ and $f''(x) < 0 \quad \Leftrightarrow \quad x < -\frac{1}{4}$. Thus, f is concave upward on $\left(-\frac{1}{4}, \infty\right)$ and concave downward on $\left(-\infty, -\frac{1}{4}\right)$. There is an inflection point at $\left(-\frac{1}{4}, f\left(-\frac{1}{4}\right)\right) = \left(-\frac{1}{4}, \frac{21}{8}\right)$.

22. (a) $f(x) = x^4(x-1)^3 \quad \Rightarrow \quad f'(x) = x^4 \cdot 3(x-1)^2 + (x-1)^3 \cdot 4x^3 = x^3(x-1)^2\,[3x + 4(x-1)] = x^3(x-1)^2(7x-4)$

The critical numbers are 0, 1, and $\frac{4}{7}$.

(b) $f''(x) = 3x^2(x-1)^2(7x-4) + x^3 \cdot 2(x-1)(7x-4) + x^3(x-1)^2 \cdot 7$

$= x^2(x-1)\,[3(x-1)(7x-4) + 2x(7x-4) + 7x(x-1)]$

Now $f''(0) = f''(1) = 0$, so the Second Derivative Test gives no information for $x = 0$ or $x = 1$.

$f''\left(\frac{4}{7}\right) = \left(\frac{4}{7}\right)^2\left(\frac{4}{7} - 1\right)\left[0 + 0 + 7\left(\frac{4}{7}\right)\left(\frac{4}{7} - 1\right)\right] = \left(\frac{4}{7}\right)^2\left(-\frac{3}{7}\right)(4)\left(-\frac{3}{7}\right) > 0$, so there is a local minimum at $x = \frac{4}{7}$.

(c) f' is positive on $(-\infty, 0)$, negative on $\left(0, \frac{4}{7}\right)$, positive on $\left(\frac{4}{7}, 1\right)$, and positive on $(1, \infty)$. So f has a local maximum at $x = 0$, a local minimum at $x = \frac{4}{7}$, and no local maximum or minimum at $x = 1$.

50. $f(x) = \dfrac{e^x}{1+e^x}$ has domain $\mathbb{R}$.

(a) $\displaystyle\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{e^x/e^x}{(1+e^x)/e^x} = \lim_{x\to\infty} \frac{1}{e^{-x}+1} = \frac{1}{0+1} = 1$, so $y = 1$ is a HA.

$\displaystyle\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{e^x}{1+e^x} = \frac{0}{1+0} = 0$, so $y = 0$ is a HA. No VA.

(b) $f'(x) = \dfrac{(1+e^x)e^x - e^x \cdot e^x}{(1+e^x)^2} = \dfrac{e^x}{(1+e^x)^2} > 0$ for all x. Thus, f is increasing on $\mathbb{R}$.

(c) There is no local maximum or minimum.

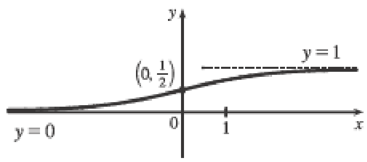
(d) $f''(x) = \dfrac{(1+e^x)^2 e^x - e^x \cdot 2(1+e^x)e^x}{[(1+e^x)^2]^2}$

$= \dfrac{e^x(1+e^x)[(1+e^x) - 2e^x]}{(1+e^x)^4} = \dfrac{e^x(1-e^x)}{(1+e^x)^3}$

$f''(x) > 0 \quad \Leftrightarrow \quad 1 - e^x > 0 \quad \Leftrightarrow \quad x < 0$, so f is CU on $(-\infty, 0)$ and CD on $(0, \infty)$.

There is an inflection point at $\left(0, \frac{1}{2}\right)$.

(e)



2. (a) $\displaystyle\lim_{x\to a} [f(x)p(x)]$ is an indeterminate form of type $0 \cdot \infty$.

(b) When x is near a, $p(x)$ is large and $h(x)$ is near 1, so $h(x)p(x)$ is large. Thus, $\displaystyle\lim_{x\to a} [h(x)p(x)] = \infty$.

(c) When x is near a, $p(x)$ and $q(x)$ are both large, so $p(x)q(x)$ is large. Thus, $\displaystyle\lim_{x\to a} [p(x)q(x)] = \infty$.

10. This limit has the form $\frac{0}{0}$. $\displaystyle\lim_{x\to 0} \frac{\sin 4x}{\tan 5x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{4\cos 4x}{5\sec^2(5x)} = \frac{4(1)}{5(1)^2} = \frac{4}{5}$

18. This limit has the form $\frac{\infty}{\infty}$. $\displaystyle\lim_{x\to\infty} \frac{\ln\ln x}{x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x\ln x} = 0$